Sparse Matrix Factorization at 1.68 Gflops

Cleve Ashcraft, Roger Grimes, John Lewis, Barry Peyton, Horst Simon, Phuong Vu

Outline

- 1988 Gordon Bell Award
- Examples of Sparse Matrices
- Standard Approach to Solving Real Symmetric Sparse Matrices
- Supernodal Approach
- Performance Comparisons

1988 Gordon Bell Award

- 1986. Research team of five Boeing employees (**Ashcraft**, Grimes, Lewis, **Peyton**, and Simon) investigated methods to improve performance of sparse matrix factorization on vector computers. Major breakthrough being the supernode concept and its application to the numeric factorization.

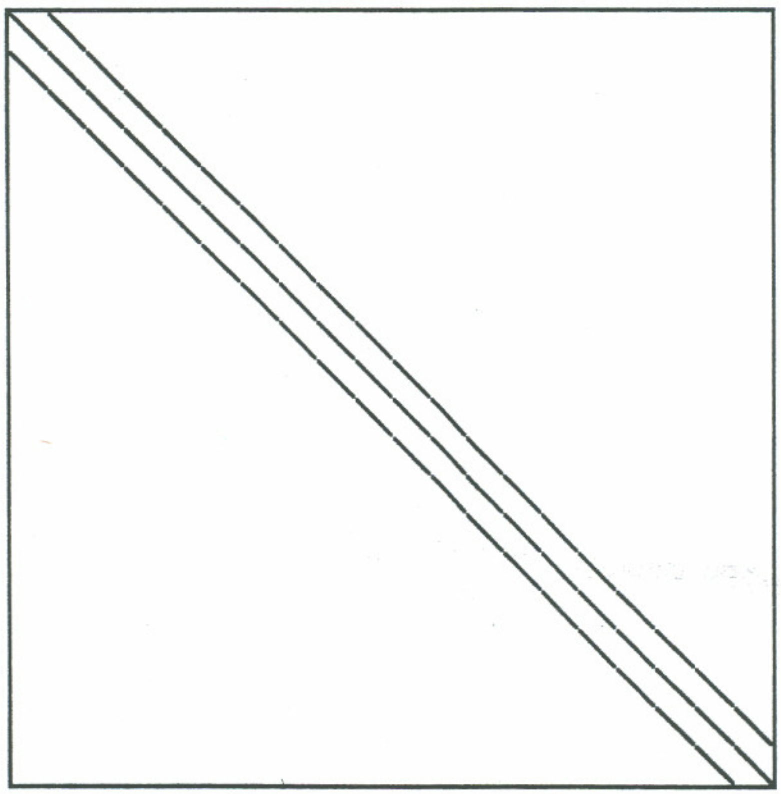
- 1987. Grimes and Peyton developed a production sparse matrix package based on the supernodal general sparse algorithm. Focused towards a single processor X-MP.

- 1988. Vu (CRI) ported software to Y-MP and multitasked it. Simon (still a Boeing employee but now supporting the NAS project at NASA Ames) utilized software on application problems. Performance exceeded 1 Gigaflop using 8 processors on Y-MP at NAS Ames. Work submitted to the Gordon Bell Award Committee.

- 1989. Gordon Bell prize for "superior efforts in practical parallel-processing research" awarded to the Boeing/CRI team.

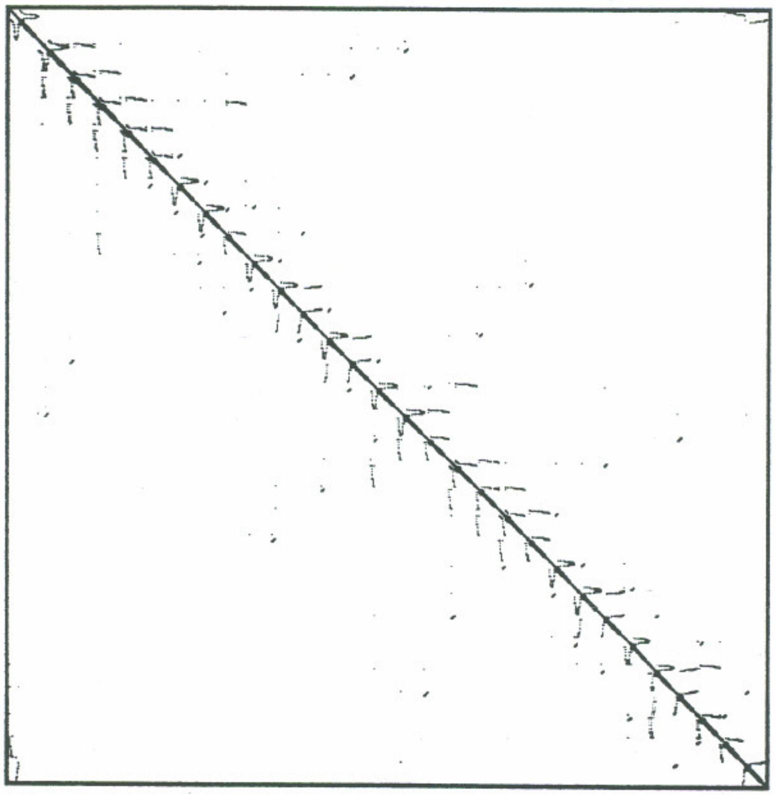
Examples of Sparse Matrices

- Arise in a wide variety of applications.
- General purpose solvers can make few assumptions about matrix properties.
- Special purpose solvers can often use properties of a specific application to great advantage.
- A collection of sparse matrices is available.



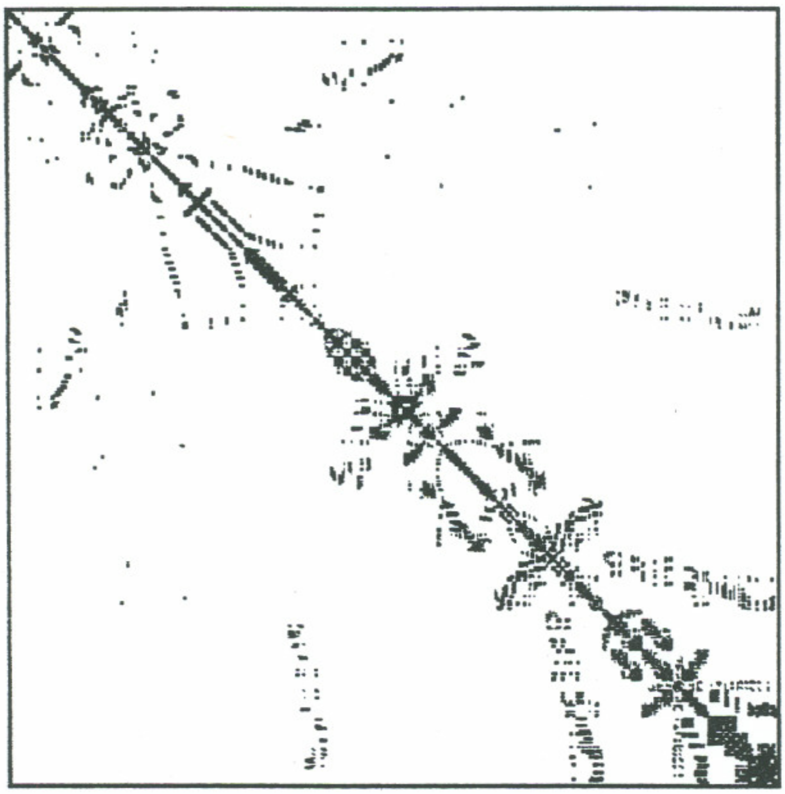
symmetric 9 point discretization of a 20 by 20 grid.

gr 20 20 400 rows 400 columns 3364 nonzeroes



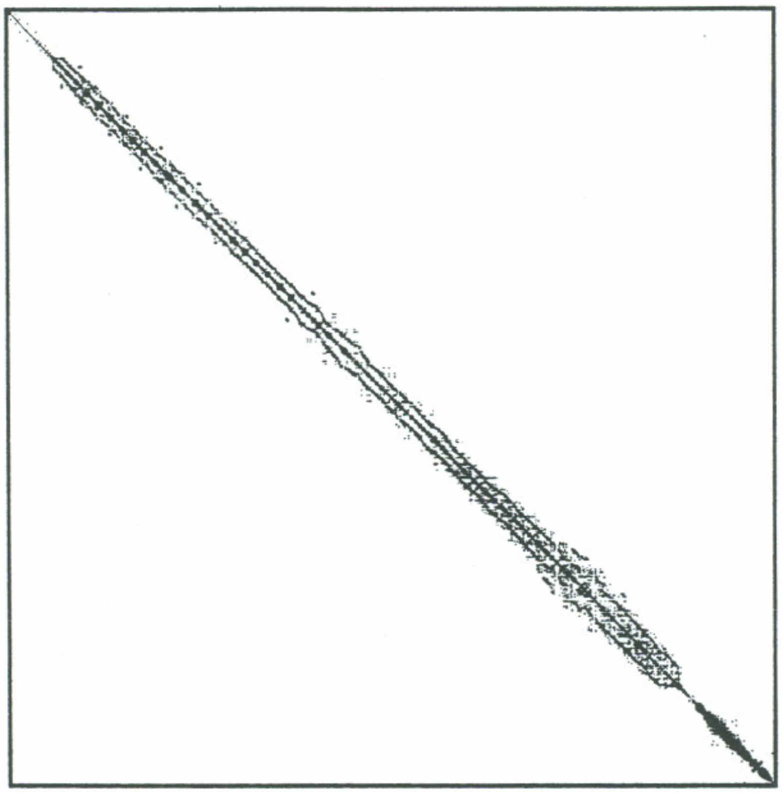
1SYMMETRIC MATRIX FROM ALAN GEORGE'S L-SHAPE PROBLEMS, 1978.

LSHP3466 3466 rows 3466 columns 23896 nonzeroes



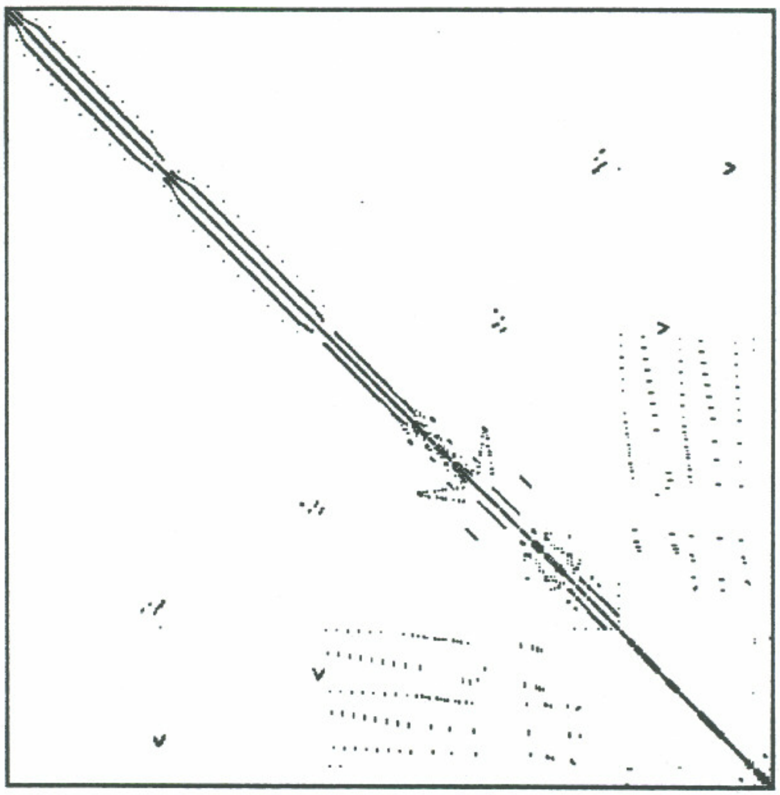
1SYMMETRIC STIFFNESS MATRIX, FLUID FLOW GENERALIZED EIGENVALUES

BCSSTK13 2003 rows 2003 columns 83883 nonzeroes



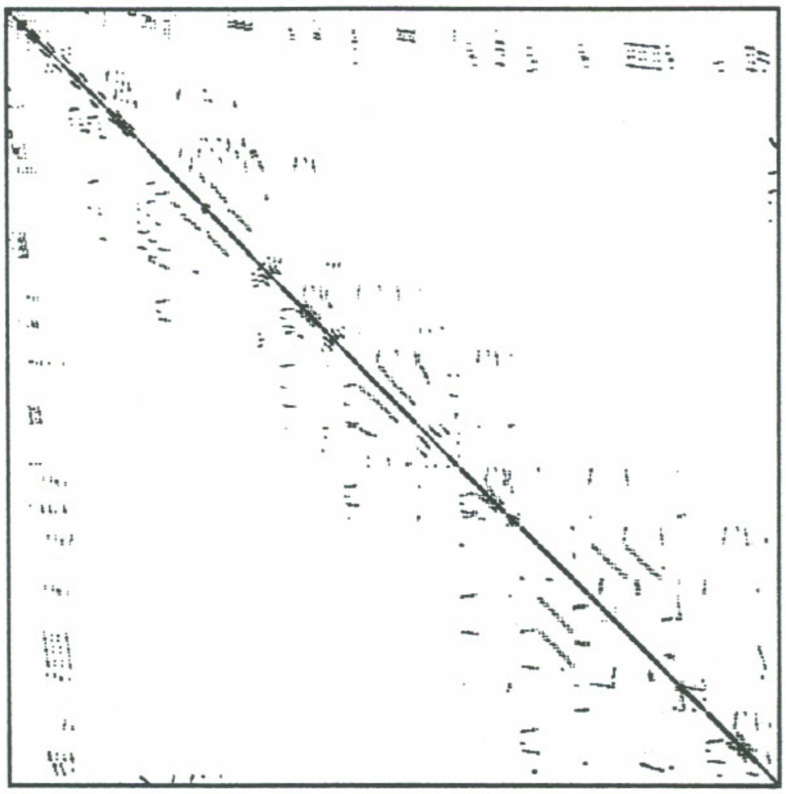
1S CONNECTIVITY STRUCTURE OF EARLY BOEING 2707 SST AIRFRAME

SSTMODEL 3345 rows 3345 columns 22749 nonzeroes



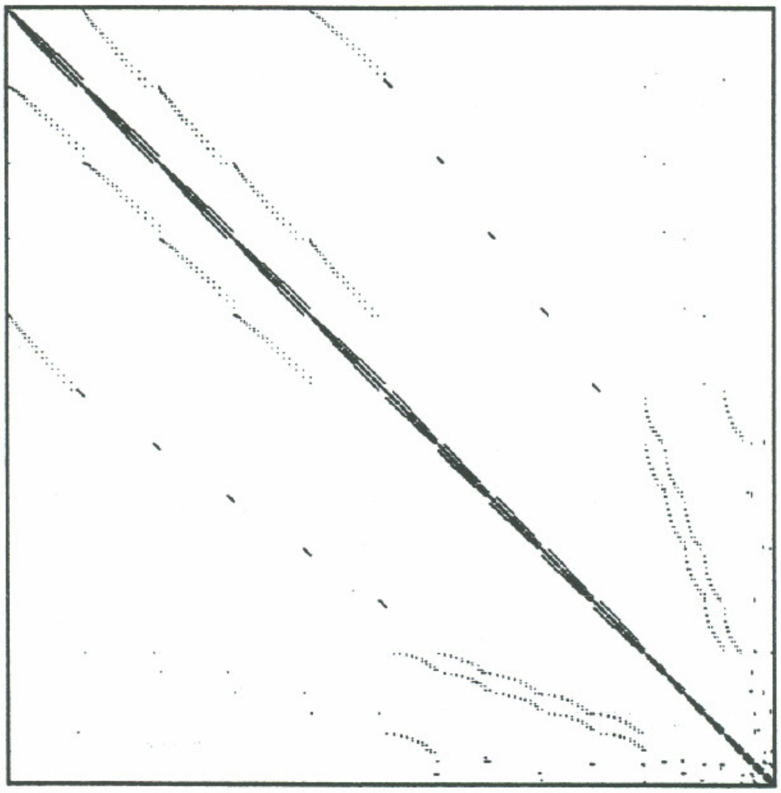
1SYMMETRIC CONNECTION TABLE FROM DTNSRDC, WASHINGTON

DWT 1242 1242 rows 1242 columns 10426 nonzeroes



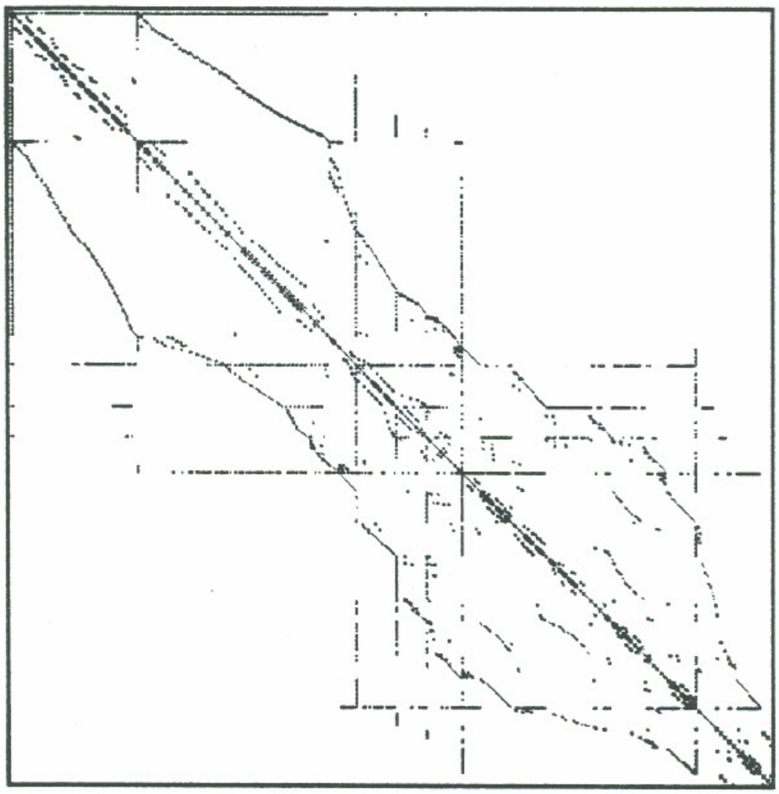
1SYMMETRIC CONNECTION TABLE FROM DTNSRDC, WASHINGTON

DWT 2680 2680 rows 2680 columns 25026 nonzeroes



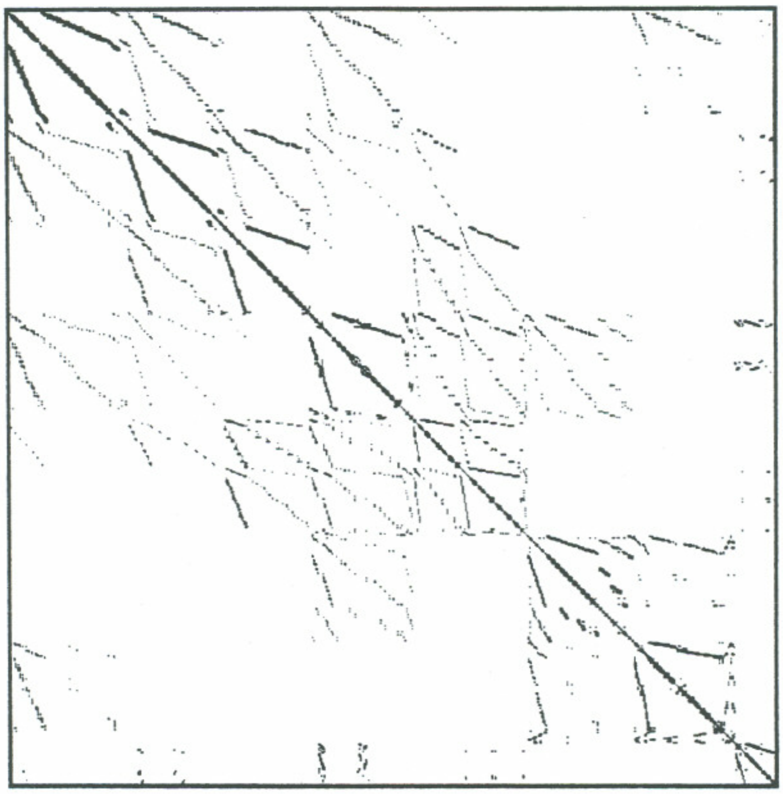
1S CONNECTIVITY STRUCTURE OF A GEODESIC DOME ON A COARSE BASE

BLCKHOLE 2132 rows 2132 columns 14872 nonzeroes



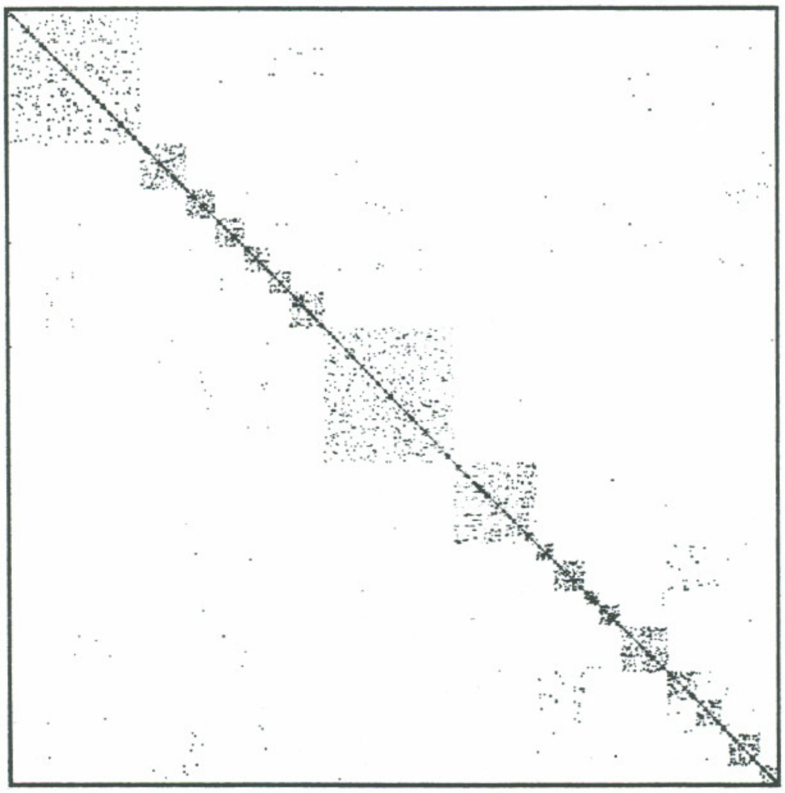
1SYMMETRIC STIFFNESS MATRIX, FRAME BUILDING (TV STUDIO)

BCSSTK08 1074 rows 1074 columns 12960 nonzeroes



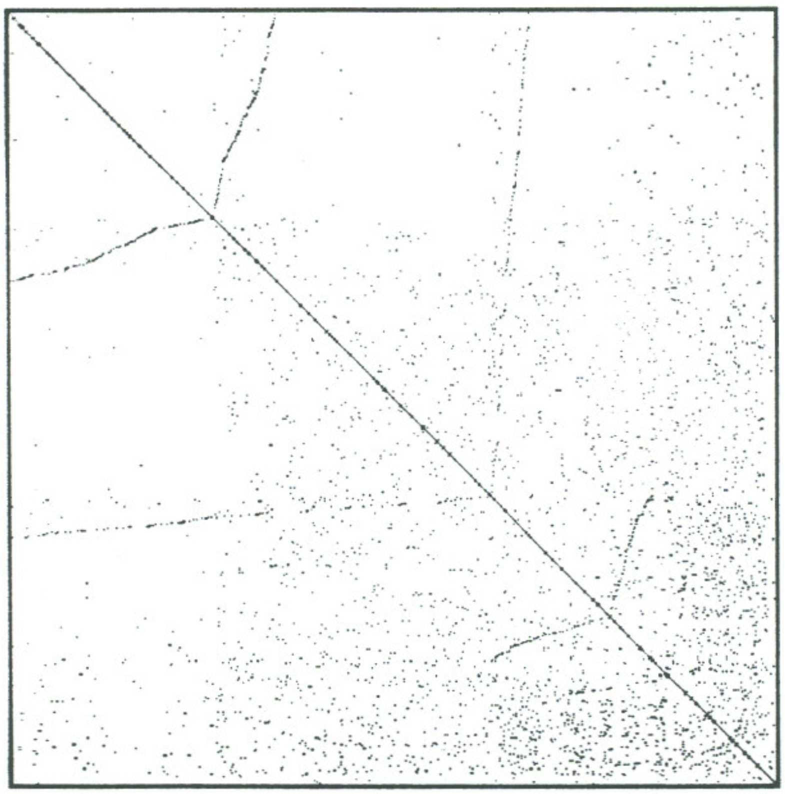
1SYMMETRIC PATTERN FROM CANNES,LUCIEN MARRO,JUNE 1981.

CAN 1072 1072 rows 1072 columns 12444 nonzeroes



1SYMMETRIC STRUCTURE REPRESENTATION OF WESTERN US POWER NETWORK

BCSPWR08 1624 rows 1624 columns 6050 nonzeroes



1SYMMETRIC STRUCTURE REPRESENTATION OF WESTERN US POWER NETWORK

BCSPWR09 1723 rows 1723 columns 6511 nonzeroes

Standard Approach for Direct Solution of Real Symnmetric Systems of Linear Equations.

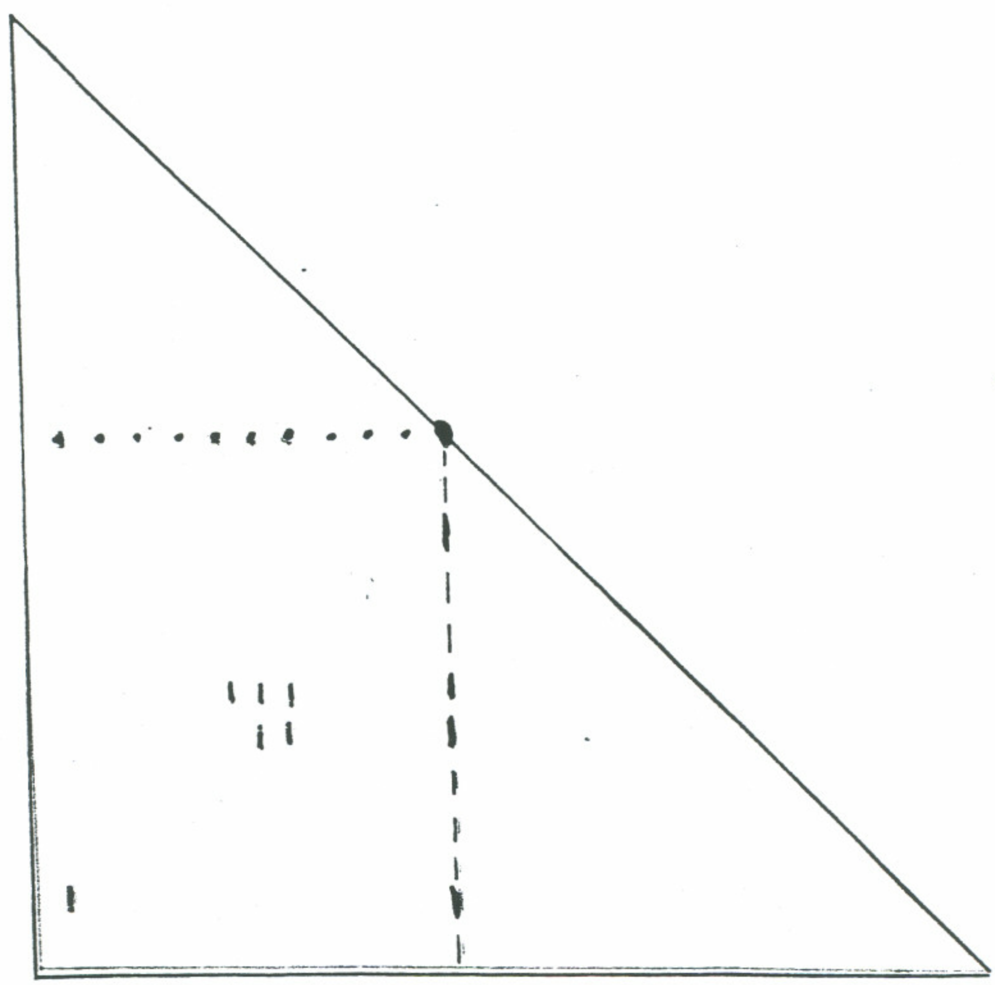
- Analyze
  - order equations to reduce cost of numeric factorization
  - build static data structure for numeric factorization
- Numeric Factorization
- Numeric Solution

ANALYZE

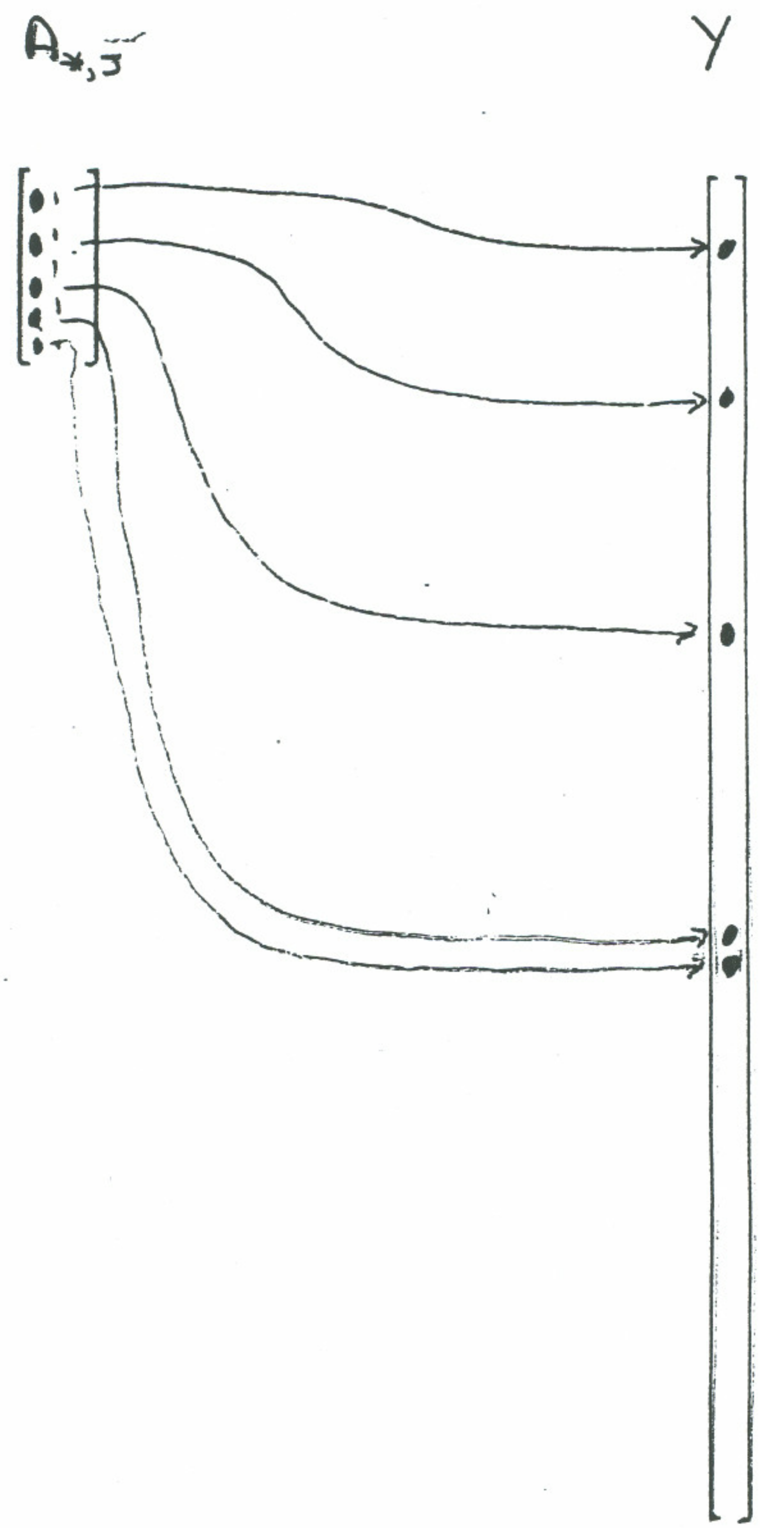
- Envelope Orderings
- Nested Dissection
- Minimum Degree
- Minimum Degree provides orderings which are
  - Reduce storage costs of the factorization by factors of 2 to 10
  - Reduce the number of floating point operations by factors of 2 to 70

SPARSE COLUMN CHOLESKY FACTORIZATION

- j-th COLUMN UPDATED BY LINEAR COMBINATION OF PREVIOUS COLUMNS

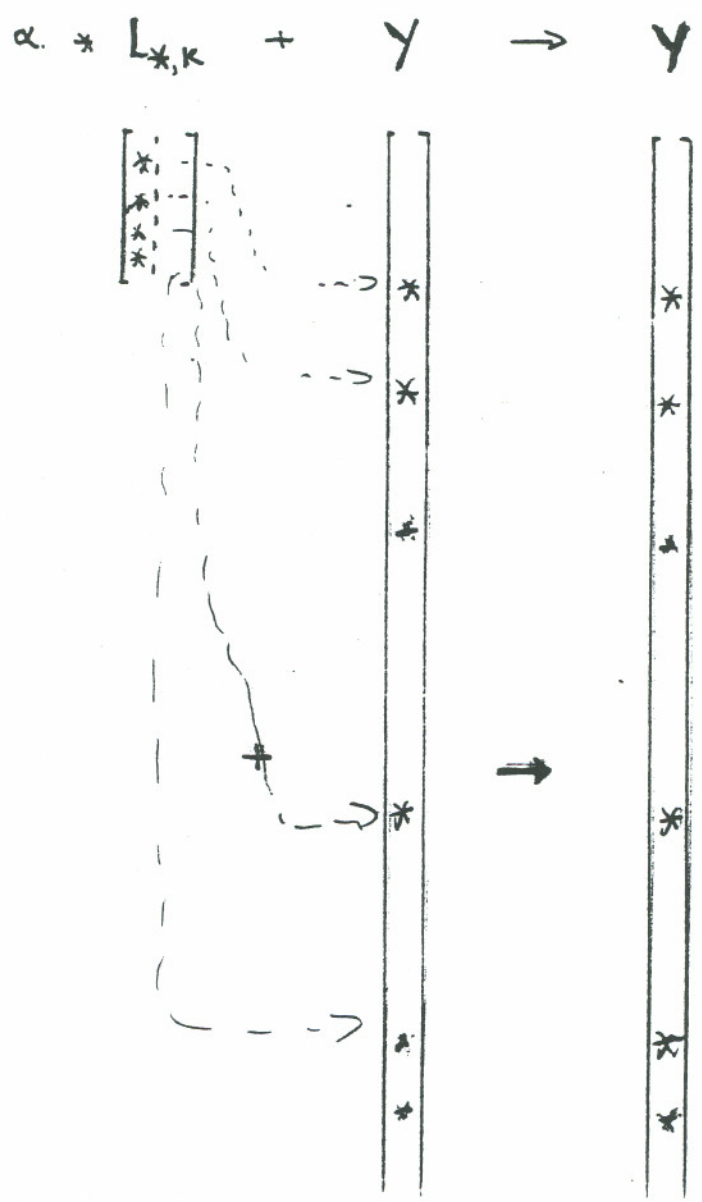


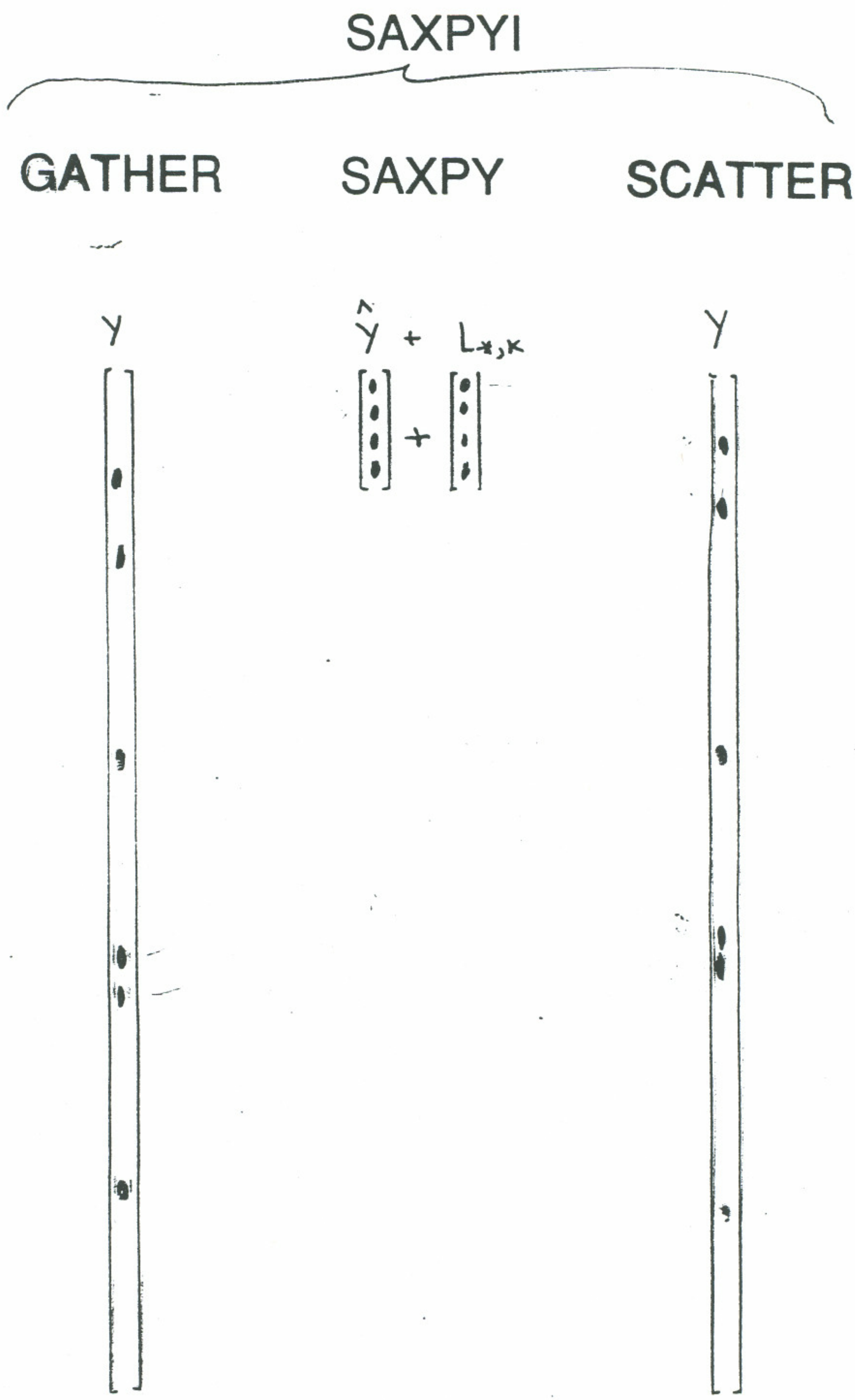
TEMPORARY EXPANSION OF THE j-th COLUMN



SAXPYI

```
DO 100  i = 1, nz
    Y(INDEX(i)) = Y(INDEX(i)) + A*X(i)
100 CONTINUE
```




SAXPYI
GATHER
SAXPY
SCATTER
Y
Ŷ + L*,K
Y

SAXPYI WITH HARDWARE GATHER/SCATTER

- o VECTOR HARDWARE *GATHERS* Y
- o VECTOR *SAXPY* TO UPDATE Y
- o VECTOR HARDWARE *SCATTERS* Y
- o 88 MEGAFLOPS ASYMPTOTIC RATE ON CRAY X-MP

SAXPYI Computational Rates on an X-MP

- For vector lengths of 10
  - 5.0 Mflops - Fortran w/out hardware gather/scatter
  - 6.3 Mflops - CAL w/out hardware gather/scatter
  - 14.0 Mflops - Fortran with hardware gather/scatter
  - 15.5 Mflops - CAL with hardware gather/scatter
- For long vector lengths
  - 5.7 Mflops - Fortran w/out hardware gather/scatter
  - 14.5 Mflops - CAL w/out hardware gather/scatter
  - 66.3 Mflops - Fortran with hardware gather/scatter
  - 88.1 Mflops - CAL with hardware gather/scatter

RELATIVE EXECUTION TIMES FOR SPARSE MATRIX FACTORIZATION

The Impact of Hardware Gather/Scatter on Sparse Gaussian Elimination (normalized so that VectorPak with g/s-1.00)

Sports Arena (N = 3562)	CFT 1.14	w/out g/s	6.3
	VectorPak	w/out g/s	2.5
	CFT 1.14	w/ g/s	1.1
	VectorPak	w/ g/s	1.0
Columbia Center (N = [redacted]) 15439	CFT 1.14	w/out g/s	8.7
	VectorPak	w/out g/s	3.1
	CFT 1.14	w/ g/s	1.2
	VectorPak	w/ g/s	1.0

BOTTLENECK IS PACKING & UNPACKING

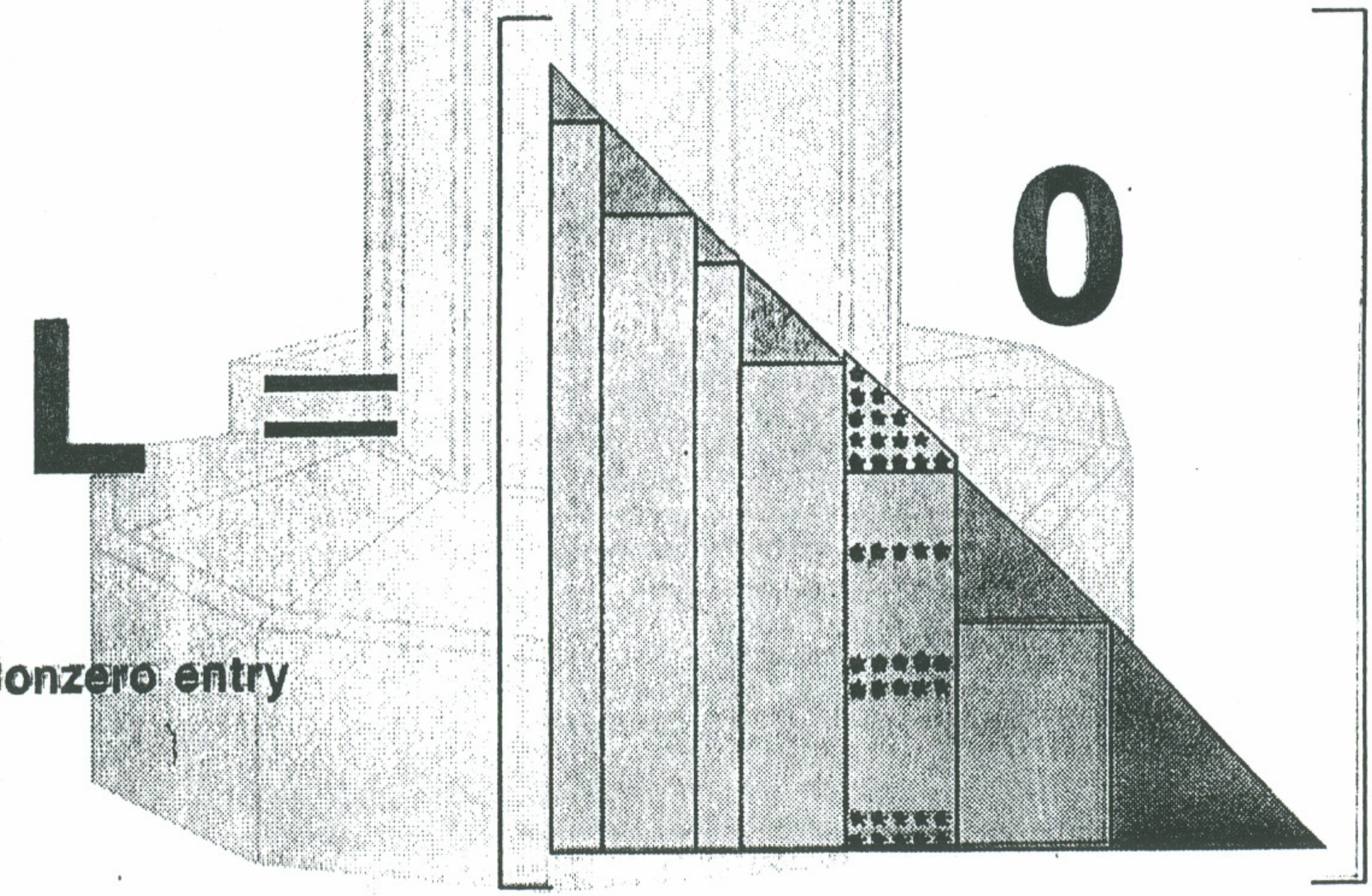
- o standard factorization updating pattern

```
SCATTER
    GATHER - SAXPY - SCATTER
    GATHER - SAXPY - SCATTER
              ...
    GATHER - SAXPY - SCATTER
GATHER
```

- o observation (Stan Eisenstat):

  often consecutive columns often have the same index pattern -- SCATTER is followed by GATHER with the same index vector

Supernode Partitioning of L



* = Nonzero entry

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SPARSE BLAS 2

UPDATING PATTERN

SCATTER

GATHER - SAXPY -
- SAXPY -
- SAXPY - SCATTER

...

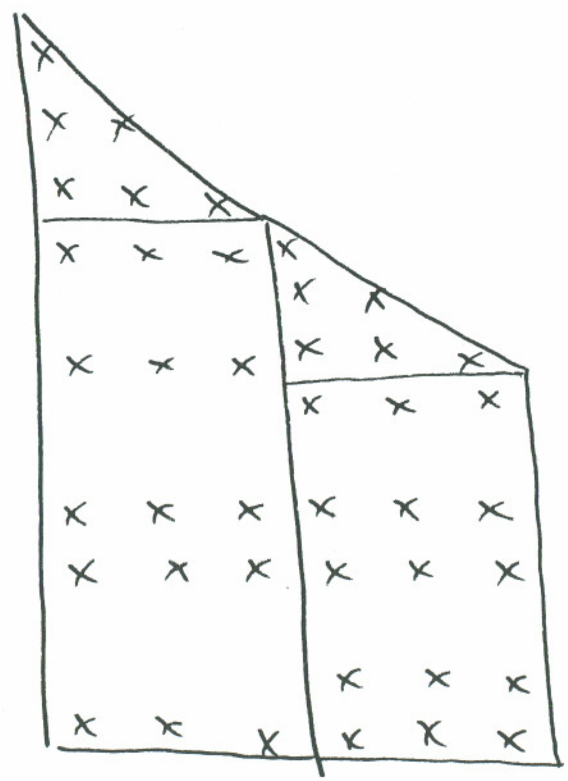
GATHER - SAXPY -
- SAXPY -
- SAXPY - SCATTER

GATHER

SAXPY
...
SAXPY

Inner Loop of Supernodal Factorization

- Sparse Matrix Multiplication where the matrix has the same column structure for every column.
- Allows Super Vector Performance



Test Problems

Label	Description	D.O.F	Average nonzeros per column	
S1	Globally triangular building	3,134	13	Small
S2	Winter sports arena	3,562	44	
S3	Offshore platform	3,948	29	
S4	Corps of Engineers dam	4,884	58	
P1	Eastern U.S. power network	5,300	3	
S5	Elevated pressure vessel	10,974	38	Medium
S6	Nuclear power station	11,948	11	
S7	Boeing 767 rear bulkhead	13,992	43	
S8	76-story skyscraper	15,439	15	
S9	Generator platform	28,924	70	Large
S10	Automobile component	35,589	32	
S11	Automobile chassis	44,609	44	

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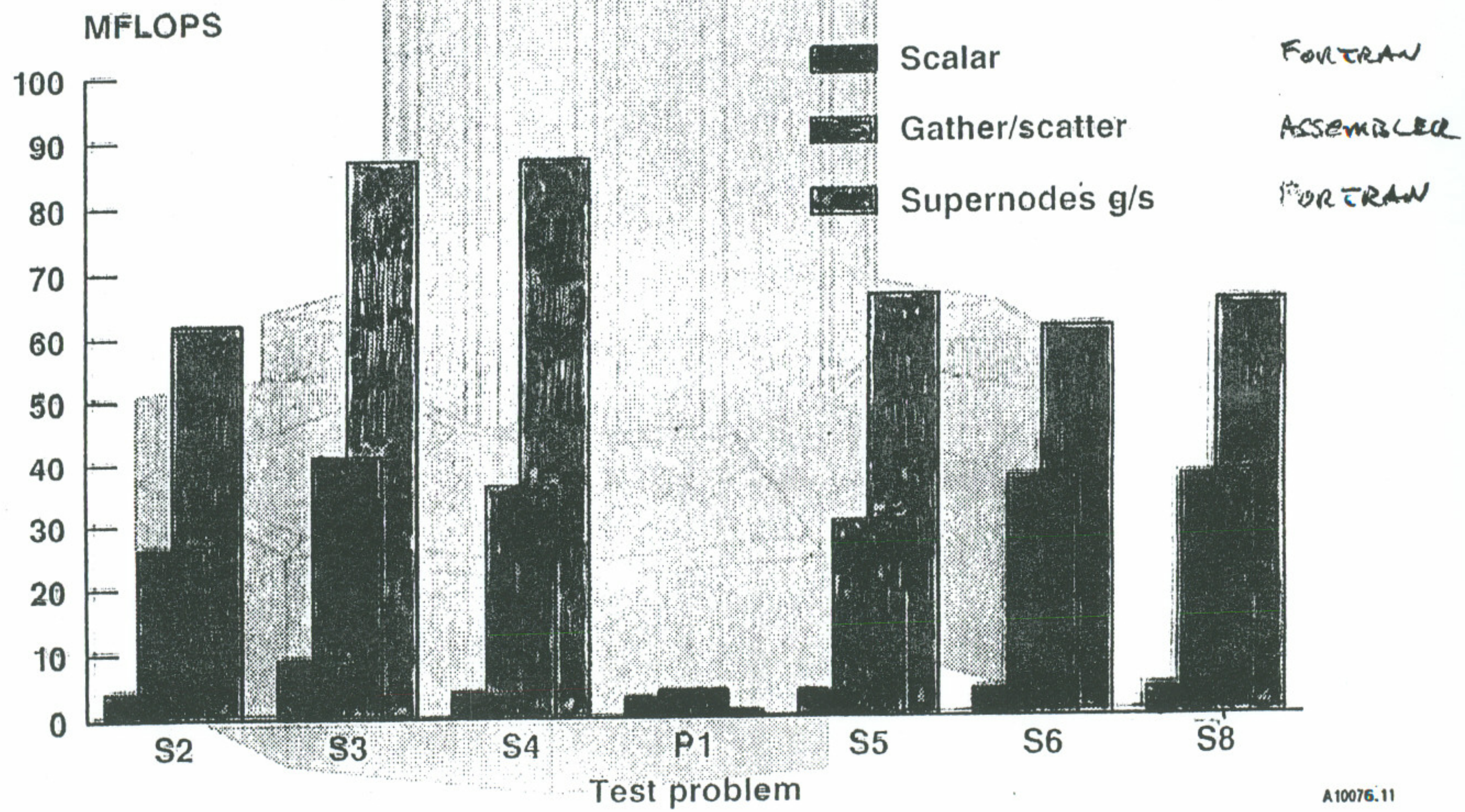
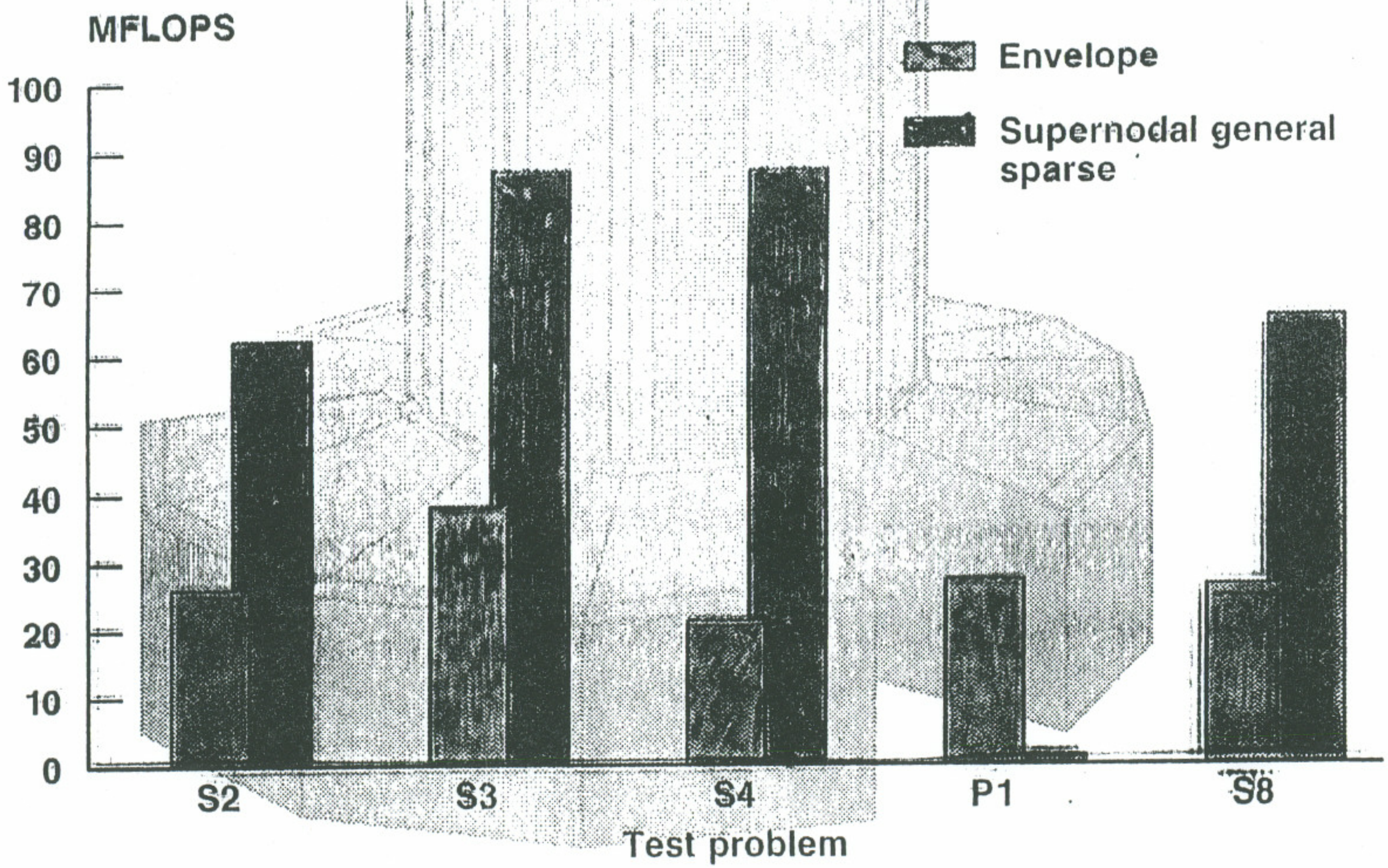
Computational Rates for General Sparse Factorization



Factorization Computational Rates
Envelope Versus Supernodal General Sparse



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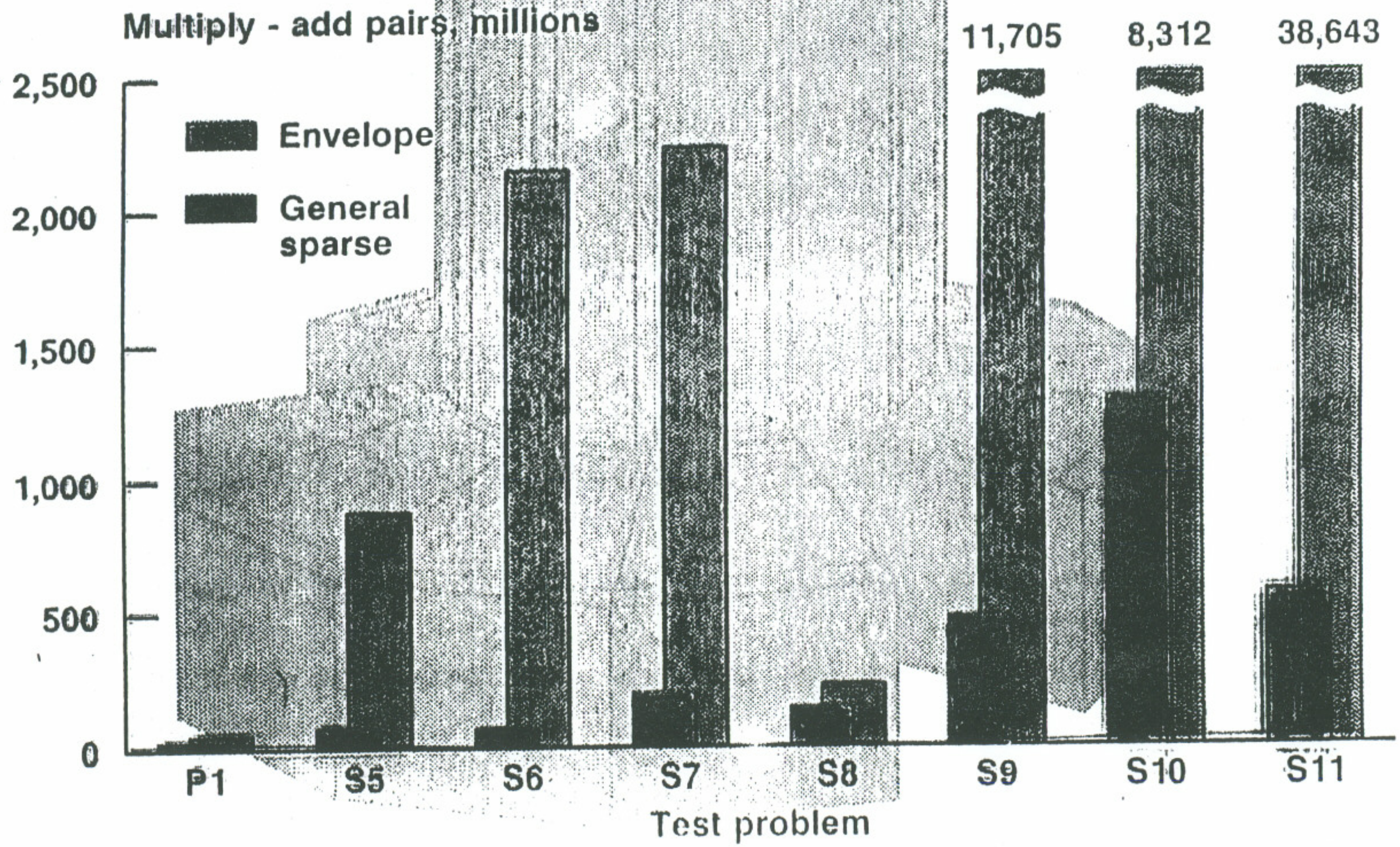
Factorization Work
Envelope Versus General Sparse



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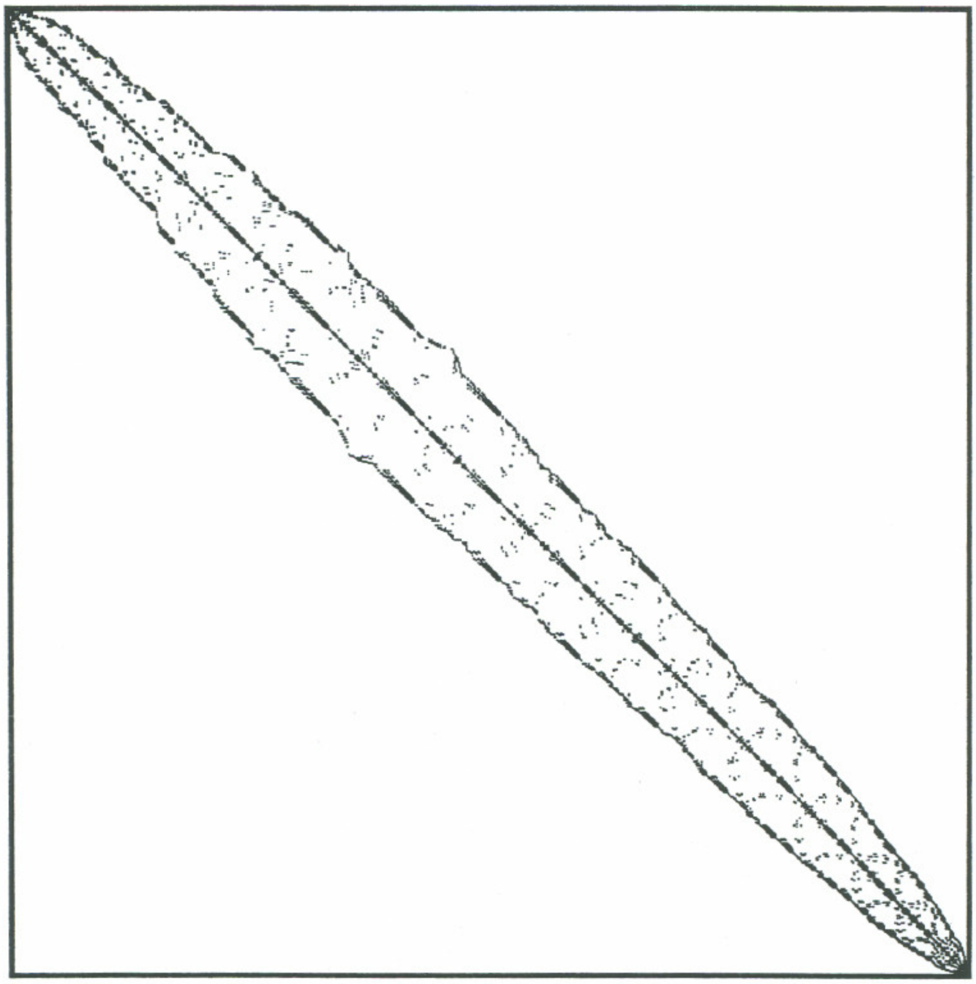
The Talk: Part II

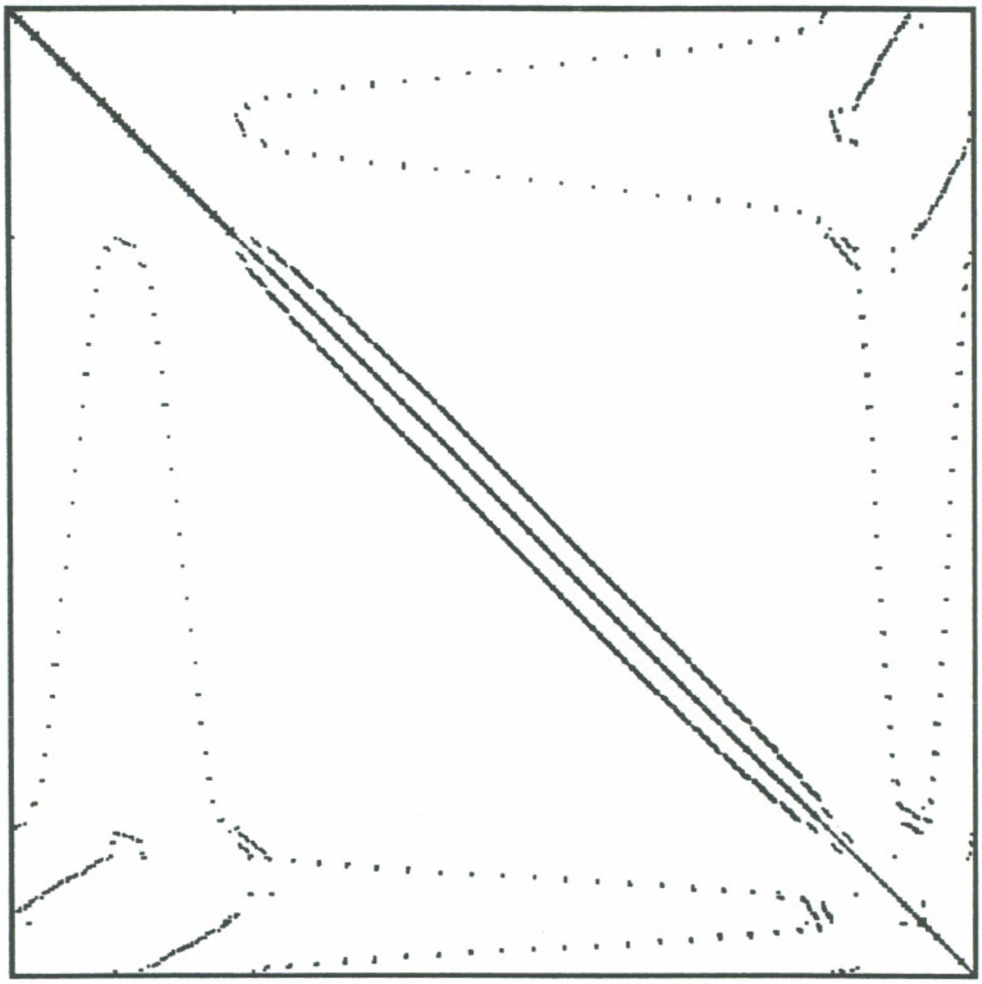
Produced by
Ken Neves

Starring in Alphabetical Order
Cleve Ashcraft
Roger Grimes
John Lewis
Barry Peyton
Horst Simon
Phuong Vu



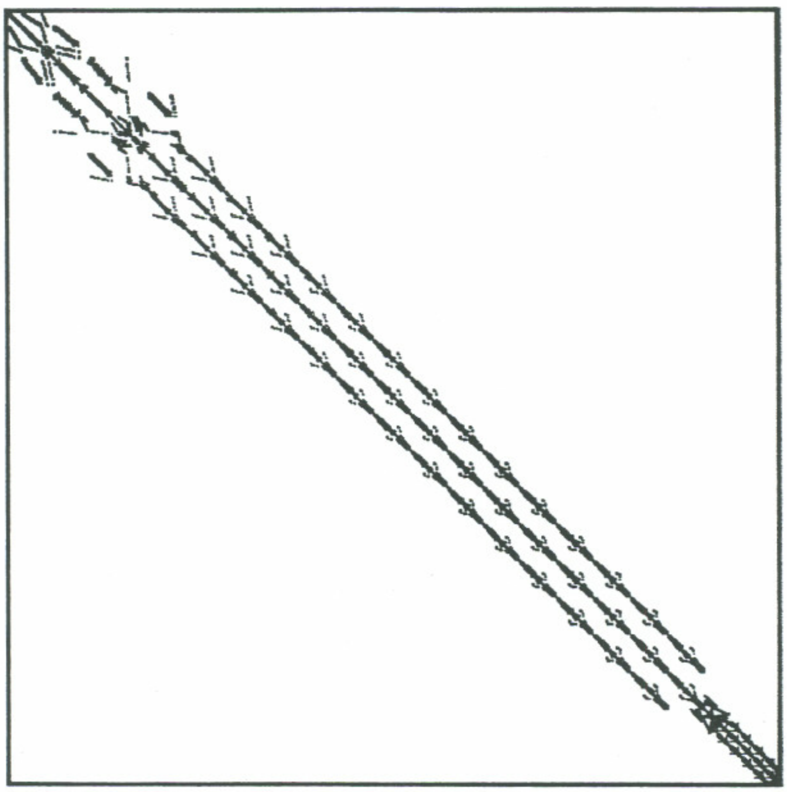
1S STIFFNESS MATRIX - MODULE OF AN OFFSHORE PLATFORM

BCSSTK15 3948 rows 3948 columns 117816 nonzeroes



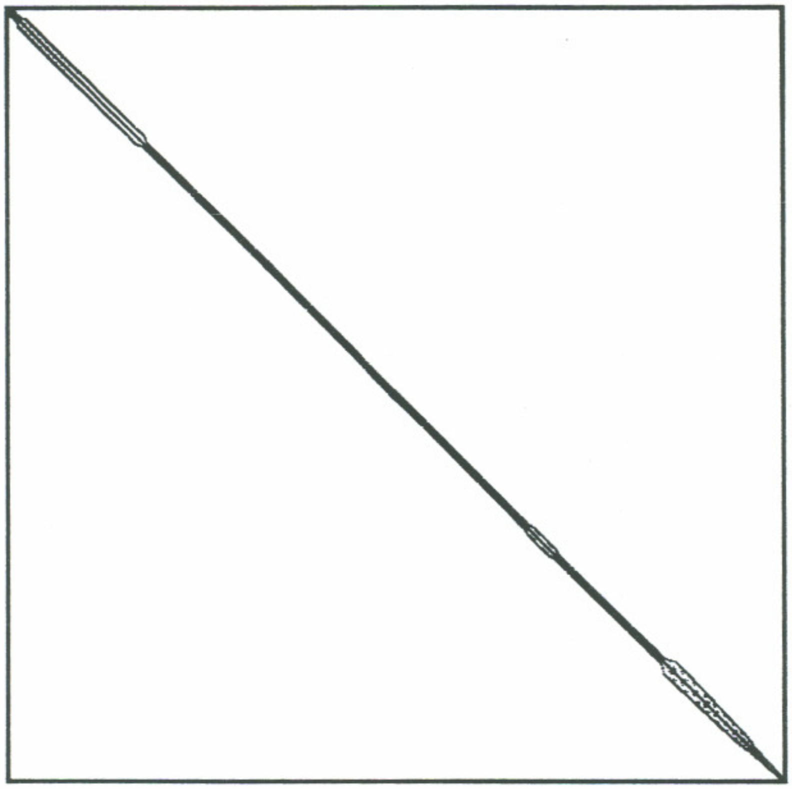
1SYMMETRIC STIFFNESS MATRIX - WINTER SPORTS ARENA

BCSSTK24 3562 rows 3562 columns 159910 nonzeroes



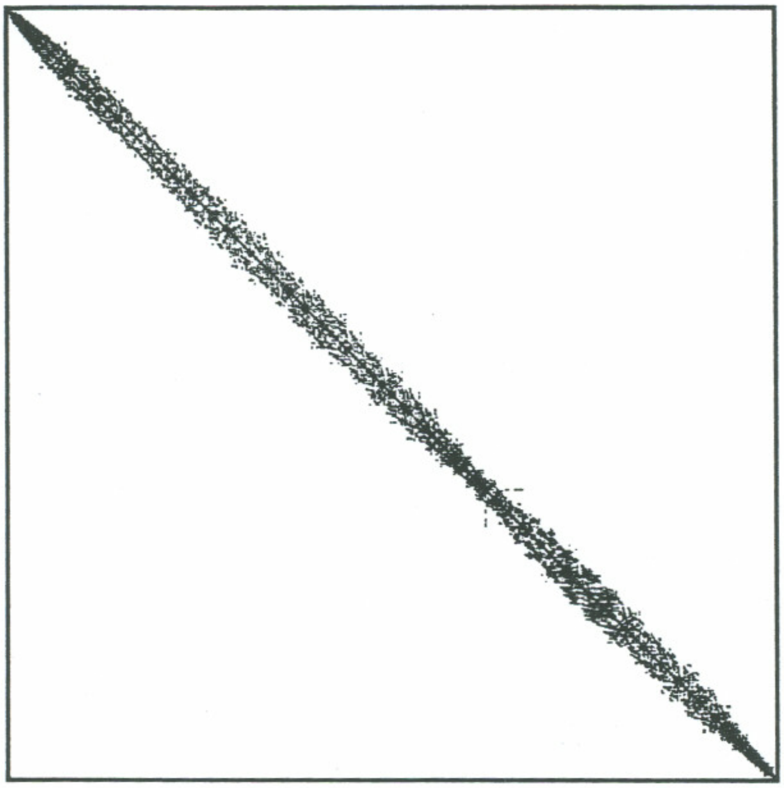
STIFFNESS MATRIX FOR PIN BOSS (AUTO STEERING COMPONENT), SOLID ELEMENTS

BCSSTK33 8738 rows 8738 columns 591904 nonzeroes



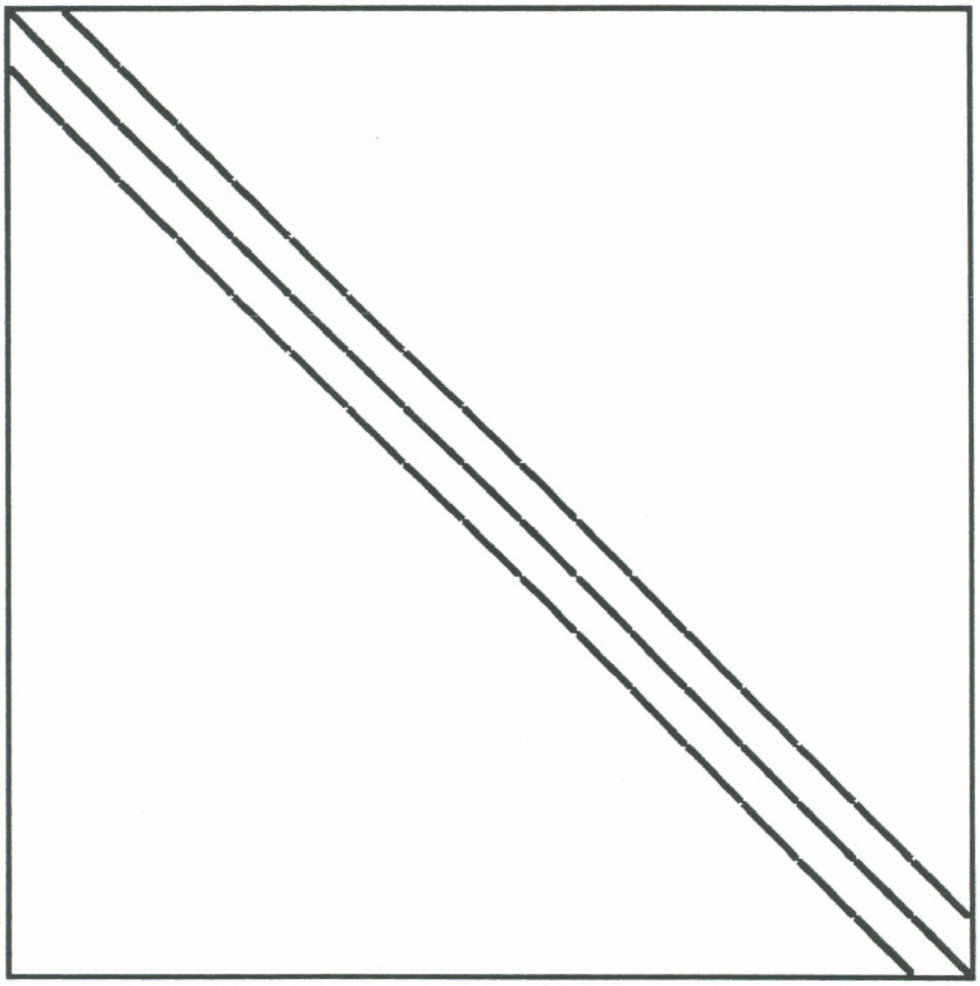
STRUCTURE FROM NASA LANGLEY, SHUTTLE ROCKET BOOSTER

NASASRB 54870 rows 54870 columns 2677324 nonzeroes



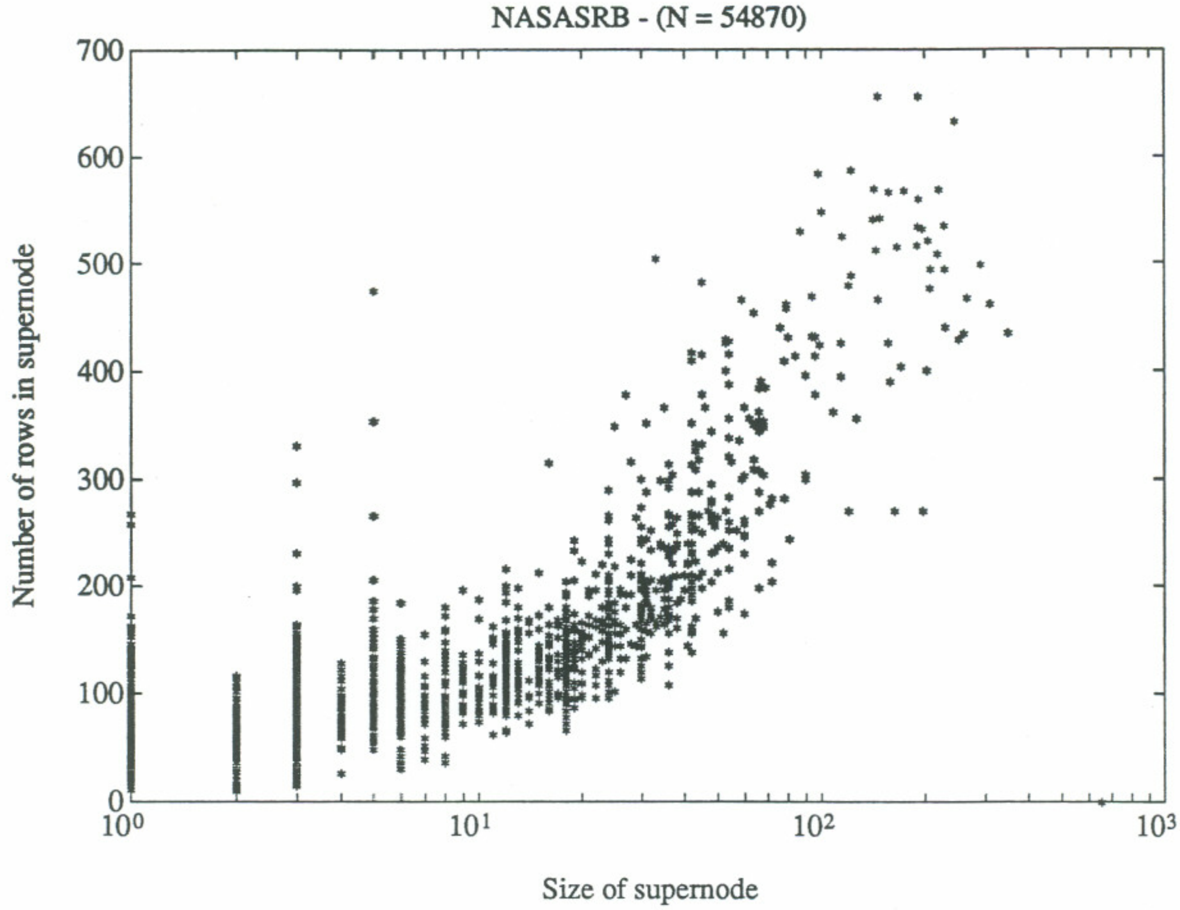
STIFFNESS MATRIX FOR AUTOMOBILE COMPONENT (MSC NASTRAN) - PATTERN ONLY

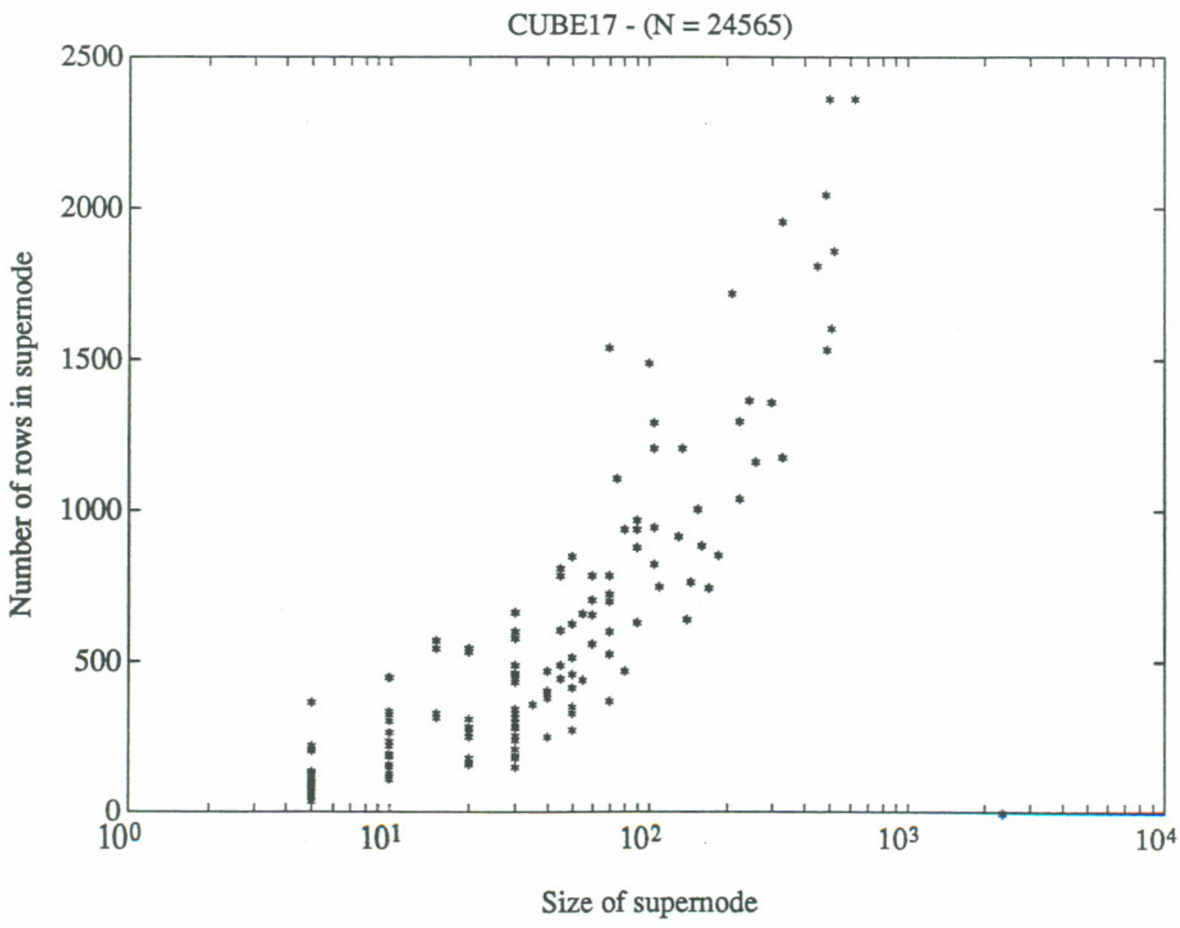
BCSSTK31 35588 rows 35588 columns 1181416 nonzeroes

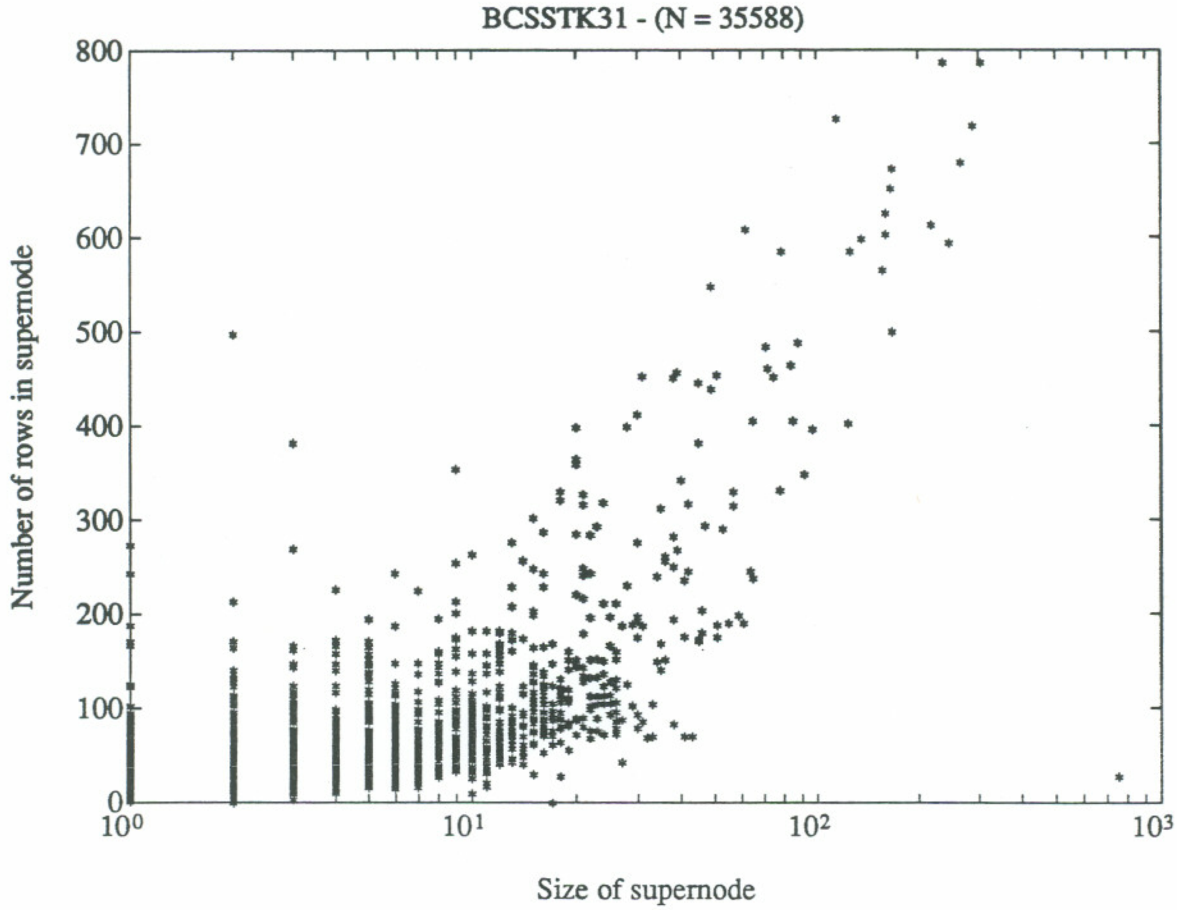


SYMMETRIC PATTERN, 27 POINT OPERATOR ON 17 BY 17 BY 17 GRID

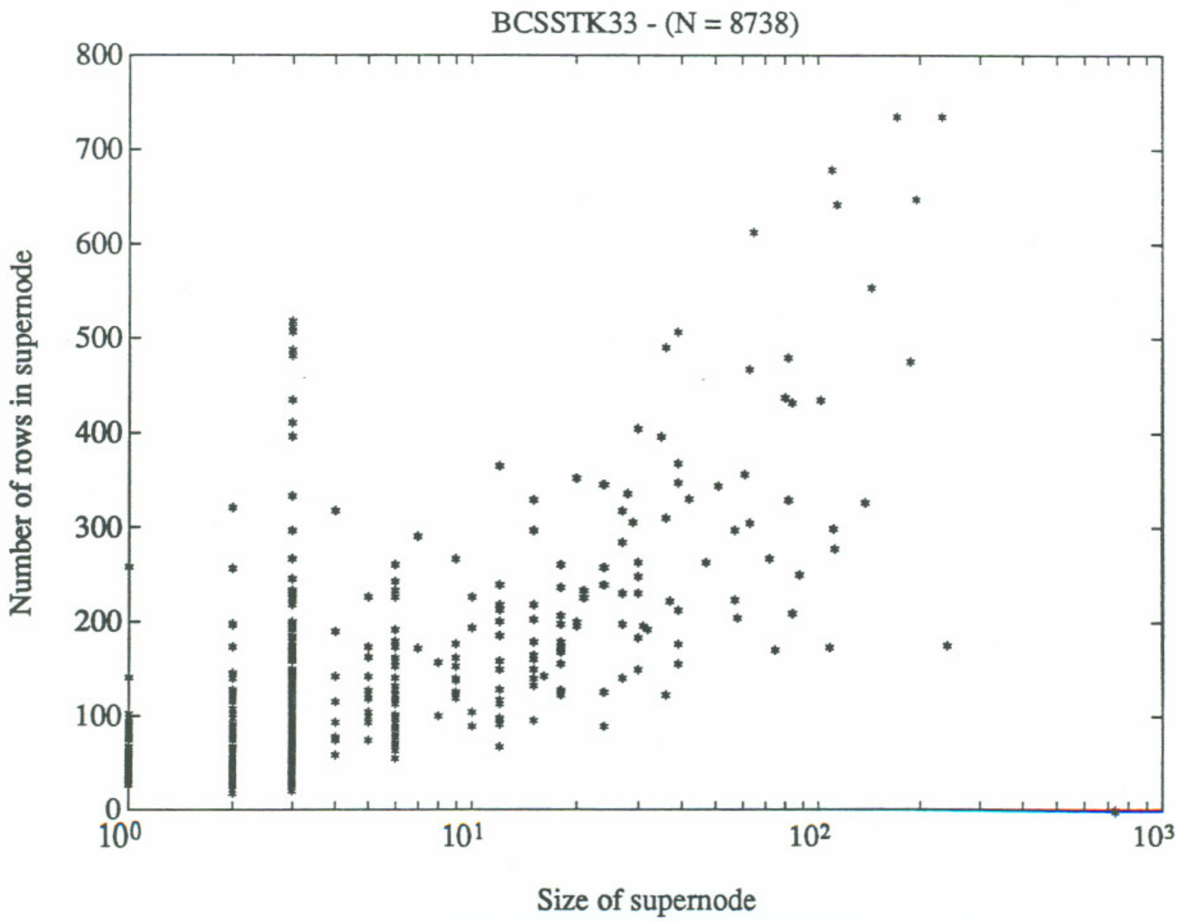
CUBE17 24565 rows 24565 columns 2941225 nonzeroes


NASASRB - (N = 54870)
Number of rows in supernode
Size of supernode
0
100
200
300
400
500
600
700
10^0
10^1
10^2
10^3

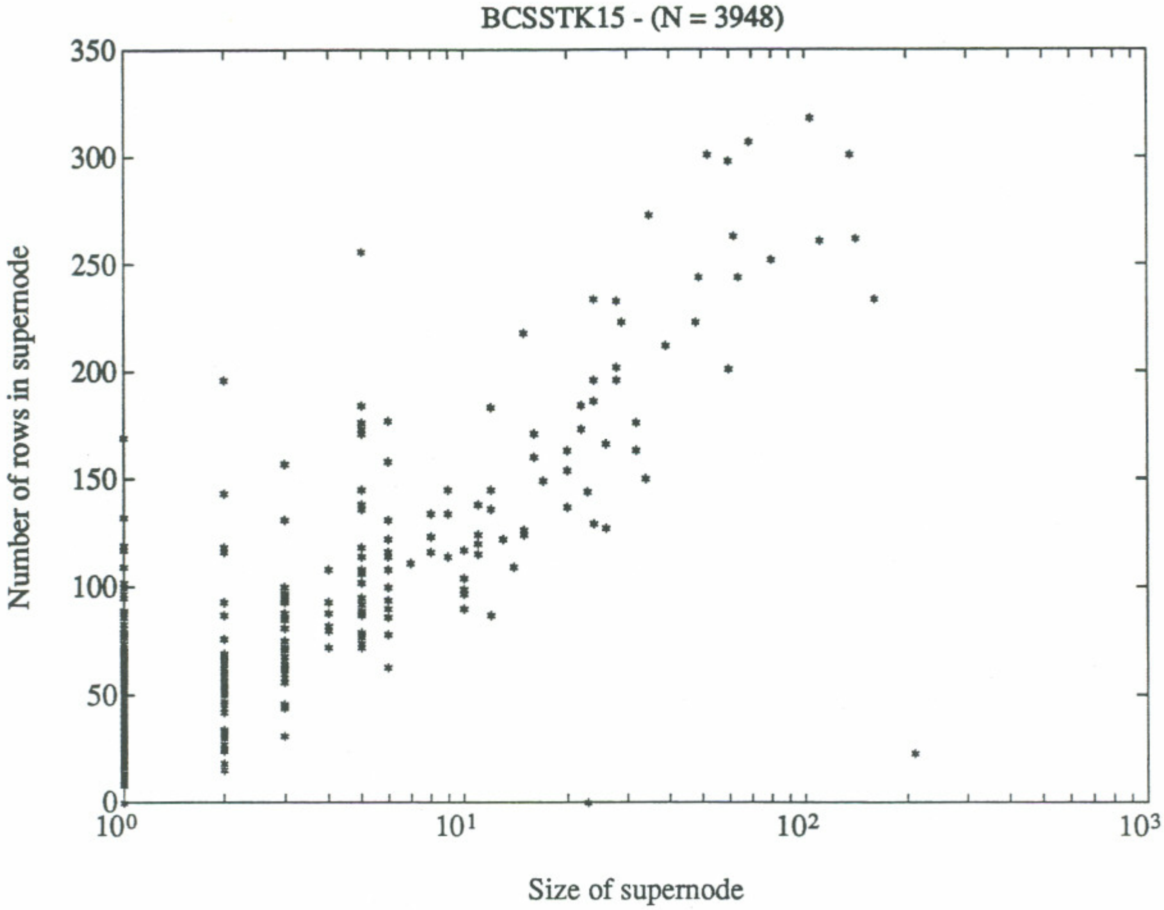

CUBE17 - (N = 24565)
Number of rows in supernode
Size of supernode
0
500
1000
1500
2000
2500
10^0
10^1
10^2
10^3
10^4

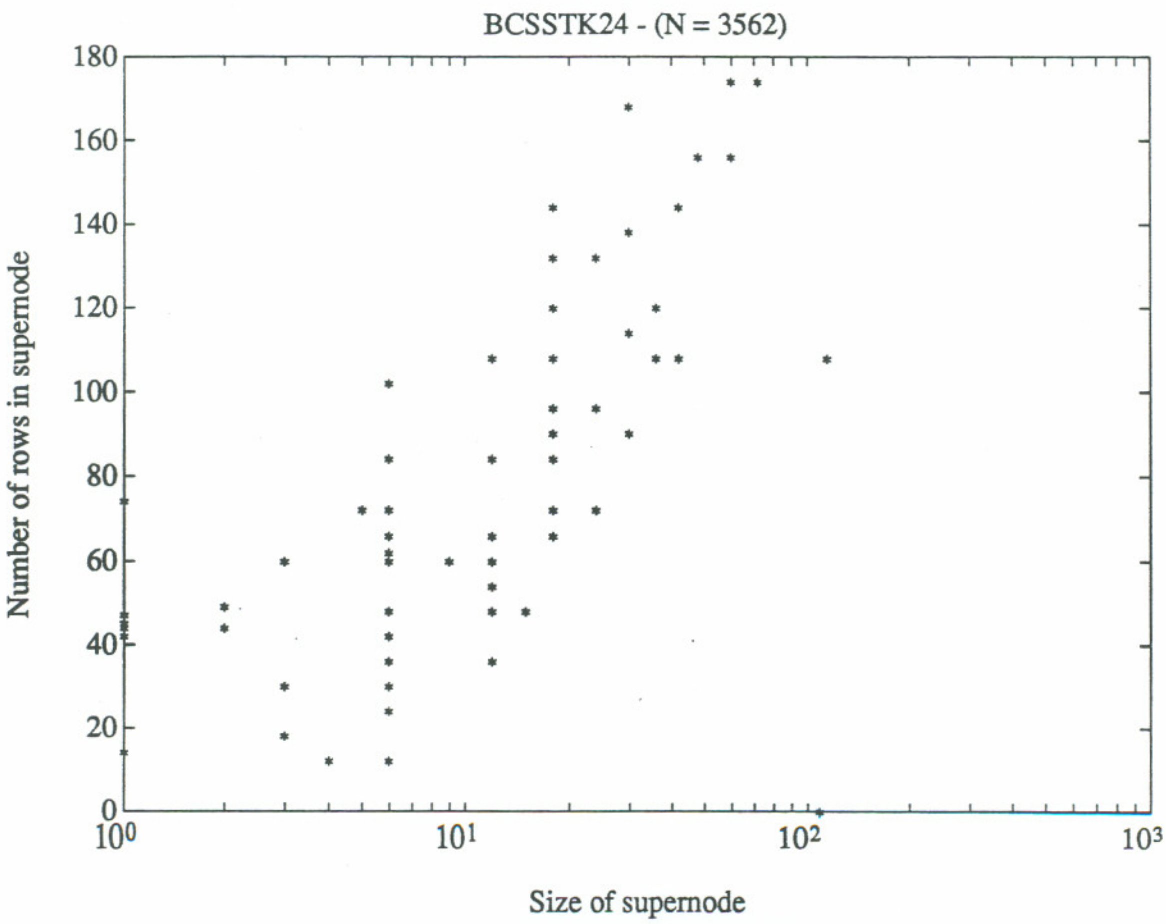


BCSSTK31 - (N = 35588)
Number of rows in supernode
Size of supernode
0
100
200
300
400
500
600
700
800
10^0
10^1
10^2
10^3

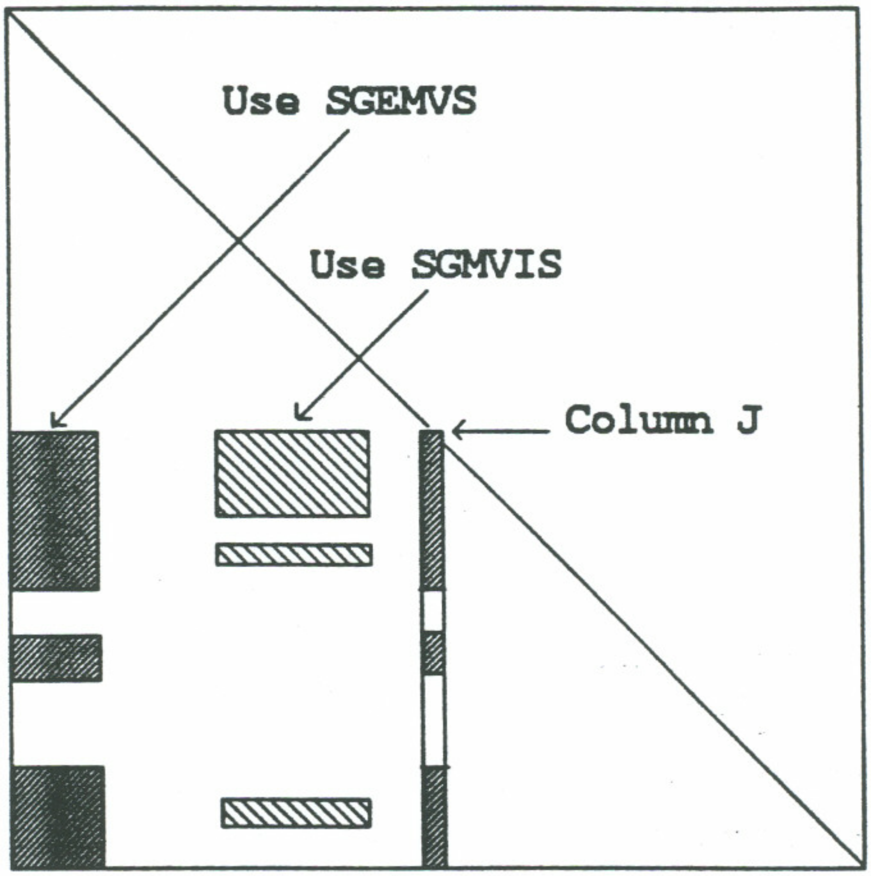


BCSSTK33 - (N = 8738)
Number of rows in supernode
Size of supernode
0
100
200
300
400
500
600
700
800
10^0
10^1
10^2
10^3


BCSSTK15 - (N = 3948)
Number of rows in supernode
Size of supernode
0
50
100
150
200
250
300
350
10^0
10^1
10^2
10^3


BCSSTK24 - (N = 3562)
Number of rows in supernode
Size of supernode
0
20
40
60
80
100
120
140
160
180
10^0
10^1
10^2
10^3

Supernode-Node Update



```
      SUBROUTINE SGEMVS
     &          (NROWS, NCOLS, A, ASTART, X, Y)
C
      INTEGER    NROWS, NCOLS, ASTART(NCOLS)
      REAL       A(*), X(NCOLS), Y(NROWS)
C
      DO  20  J = 1, NCOLS
          K1 = ASTART(J)
CDIR$ IVDEP
          DO  10  I = 1, NROWS
              Y(I) = Y(I) - X(J) * A(K1)
              K1   = K1 + 1
   10     CONTINUE
   20 CONTINUE
C
      RETURN
      END
```

```
      SUBROUTINE SGMVIS
     &           (NROWS, NCOLS, A, ASTART, X, INDEX, Y)
C
      INTEGER    NROWS, NCOLS, ASTART(NCOLS),
     &           INDEX(NROWS)
      REAL       A(*), X(NCOLS), Y(NROWS)
C
      DO  20  J = 1, NCOLS
          K1 = ASTART(J)
CDIR$ IVDEP
          DO  10  II = 1, NROWS
              I     = INDEX(II)
              Y(I)  = Y(I) - X(J) * A(K1)
              K1    = K1 + 1
   10     CONTINUE
   20 CONTINUE
C
      RETURN
      END
```

```
      DO  20  J = JMIN, JMAX, 8
          K0 = ASTART(J)
          K1 = ASTART(J-1)
          K2 = ASTART(J-2)
          K3 = ASTART(J-3)
          K4 = ASTART(J-4)
          K5 = ASTART(J-5)
          K6 = ASTART(J-6)
          K7 = ASTART(J-7)
          DO  10  I = 1, NROWS
              Y(I) = (((((((( Y(I) )
     &           - X(J  ) * A(K0) ) - X(J-1) * A(K1) )
     &           - X(J-2) * A(K2) ) - X(J-3) * A(K3) )
     &           - X(J-4) * A(K4) ) - X(J-5) * A(K5) )
     &           - X(J-6) * A(K6) ) - X(J-7) * A(K7)
              K0 = K0 + 1
              K1 = K1 + 1
              K2 = K2 + 1
              K3 = K3 + 1
              K4 = K4 + 1
              K5 = K5 + 1
              K6 = K6 + 1
              K7 = K7 + 1
   10     CONTINUE
   20 CONTINUE
```

```
      DO  20  J = JMIN, JMAX, 8
          K0 = ASTART(J)
          K1 = ASTART(J-1)
          K2 = ASTART(J-2)
          K3 = ASTART(J-3)
          K4 = ASTART(J-4)
          K5 = ASTART(J-5)
          K6 = ASTART(J-6)
          K7 = ASTART(J-7)
          DO  10  II = 1, NROWS
              I    = INDEX(II)
              Y(I) = (((((((( Y(I) )
     &           - X(J  ) * A(K0) ) - X(J-1) * A(K1) )
     &           - X(J-2) * A(K2) ) - X(J-3) * A(K3) )
     &           - X(J-4) * A(K4) ) - X(J-5) * A(K5) )
     &           - X(J-6) * A(K6) ) - X(J-7) * A(K7)
              K0 = K0 + 1
              K1 = K1 + 1
              K2 = K2 + 1
              K3 = K3 + 1
              K4 = K4 + 1
              K5 = K5 + 1
              K6 = K6 + 1
              K7 = K7 + 1
 10       CONTINUE
 20   CONTINUE
```

Numerical factorization results
Single CPU Cray YMP - running 6.49ns

Problem	Neqns	NumFact with SGEMVS+SGMVIS Time	NumFact with SGEMVS+SGMVIS Mflops	NumFact with SGEMV8+SGMVI8 Time	NumFact with SGEMV8+SGMVI8 Mflops	NumFact with Inline MV+MVI Time	NumFact with Inline MV+MVI Mflops
BCSSTK15	3948	0.91	182.81	1.10	150.45	1.90	87.03
BCSSTK24	3562	0.25	128.86	0.33	97.97	0.49	66.26
BCSSTK31	35588	11.00	232.38	13.22	193.31	24.42	104.69
BCSSTK33	8738	5.05	238.86	6.05	199.11	11.15	108.21
NASASRB	54870	20.21	231.71	24.99	187.45	48.08	104.64
CUBE17	24565	115.24	280.75	134.20	241.07	259.52	124.66
EG-AAT	25302	31.04	268.87				

Blending model
from GE. Matrix from Karmarkar's algorithm

From *perftrace* most of the time in the numerical factorization phase is in the MV and MVI routines $\Longrightarrow$ good candidate for autotasking. Possible choices strategies:

- Use inline code
- Use single tasked CAL kernel
- Autotasking by rows
- Autotasking by columns

Instructions Issued	MIPS = 20.79	1301443901	7.41 Average CP per instruction
Clock periods holding issue		8233112729	85.34% of CP holding issue
Instruction Buffer Fetches	0.00	0	
I/O memory references	0.07	4089969	0.04% of total references
CPU memory references	166.07	10398539915	0.63 Memory references per FLO
Floating point add operations	132.41	8291027471	50.01% of total FLOPS
Floating point multiply operations	132.35	8287215138	49.99% of total FLOPS
Floating point reciprocal operations	0.00	0	0.00% of total FLOPS
Total floating point operations	MFLOPS = 264.77	16578242609	51.19% of program FLOPS

Whole program (including all routines) Accounted CP = 22656900009 CPU seconds = 147.04328106

Description	Million per second	Number	
Instructions Issued	MIPS = 22.98	3378328217	6.71 Average CP per instruction
Clock periods holding issue		18607809818	82.13% of CP holding issue
Instruction Buffer Fetches	0.04	5816867	
I/O memory references	0.06	8100767	0.04% of total references
CPU memory references	148.94	21900519919	0.68 Memory references per FLO
Floating point add operations	110.17	16200150970	50.02% of total FLOPS
Floating point multiply operations	110.07	16184989409	49.98% of total FLOPS
Floating point reciprocal operations	0.00	376755	0.00% of total FLOPS
Total floating point operations	MFLOPS = 220.24	32385517134	100.00% of program FLOPS

********* Group 0 - Condensed output for all routines *********

Rank	Order	Name	Times called	Time in seconds	Average Time	Execute Percent	Accum Percent	Adds	Mults	Recip	FLOPS	Memory /FLOP	Millio Mem/Se
1	20	SGMVI8	279705	6.69E+01	2.39E-04	45.53	45.53	7.88E+09	7.83E+09	0.00E+00	1.57E+10	0.70	163.7
2	19	SGEMV8	20859	6.26E+01	3.00E-03	42.58	88.11	8.29E+09	8.29E+09	0.00E+00	1.66E+10	0.63	166.0
3	18	GSICF2	1	4.89E+00	4.89E+00	3.33	91.44	2.14E+06	4.39E+07	3.24E+05	4.64E+07	5.88	55.7
4	16	INPVAL	1	3.70E+00	3.70E+00	2.52	93.96	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00	29.2
5	7	MMDELM	1483	2.99E+00	2.01E-03	2.03	95.99	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00	6.2
6	8	MMDUPD	115	1.75E+00	1.52E-02	1.19	97.17	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00	6.1
7	2	LTFADJ	1	1.43E+00	1.43E+00	0.97	98.15	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00	12.4
8	14	ADJTRI	1	1.19E+00	1.19E+00	0.81	98.96	1.72E+05	9.80E+04	2.45E+04	2.94E+05	36.28	8.9
9	1	SSSISM	1	5.12E-01	5.12E-01	0.35	99.30	4.70E+01	0.00E+00	0.00E+00	4.70E+01	64156.02	5.8
10	12	SMBFAC	1	3.51E-01	3.51E-01	0.24	99.54	3.79E+03	4.56E+03	9.44E+02	9.29E+03	495.69	13.1
11	25	GSBSLV	1	3.32E-01	3.32E-01	0.23	99.77	2.25E+07	2.07E+07	0.00E+00	4.32E+07	1.51	195.8
12	13	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

Description	Million per second	Number	
Instructions Issued	MIPS = 47.01	29385016	3.28 Average CP per instruction
Clock periods holding issue		53605888	55.66% of CP holding issue
Instruction Buffer Fetches	0.00	8	
I/O memory references	0.01	9087	0.06% of total references
CPU memory references	23.25	14532795	0.00 Memory references per FLO
Floating point add operations	0.00	0	0.00% of total FLOPS
Floating point multiply operations	0.00	0	0.00% of total FLOPS
Floating point reciprocal operations	0.00	0	0.00% of total FLOPS
Total floating point operations	MFLOPS = 0.00	0	0.00% of program FLOPS

Whole program (including all routines)   Accounted CP =   1602023345   CPU seconds =   10.39713151

Description	Million per second	Number	
Instructions Issued	MIPS = 33.56	348912015	4.59 Average CP per instruction
Clock periods holding issue		1089620603	68.02% of CP holding issue
Instruction Buffer Fetches	0.18	1842341	
I/O memory references	0.80	8279722	0.89% of total references
CPU memory references	88.66	921818762	0.77 Memory references per FLO
Floating point add operations	58.35	606706779	50.44% of total FLOPS
Floating point multiply operations	57.33	596113076	49.56% of total FLOPS
Floating point reciprocal operations	0.01	110590	0.01% of total FLOPS
Total floating point operations	MFLOPS = 115.70	1202930445	100.00% of program FLOPS

********* Group 0 - Condensed output for all routines *********

Rank	Order	Name	Times called	Time in seconds	Average Time	Execute Percent	Accum Percent	Adds	Mults	Recip	FLOPS	Memory /FLOP	Millio Mem/Se
1	20	SGMVI8	75223	2.40E+00	3.19E-05	23.10	23.10	3.08E+08	2.94E+08	0.00E+00	6.02E+08	0.74	185.6
2	19	SGEMV8	6889	2.27E+00	3.29E-04	21.81	44.91	2.92E+08	2.91E+08	0.00E+00	5.83E+08	0.63	163.0
3	8	MMDUPD	281	1.85E+00	6.59E-03	17.82	62.73	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00	6.5
4	18	GSICF2	1	1.40E+00	1.40E+00	13.48	76.21	2.46E+06	7.63E+06	9.05E+04	1.02E+07	5.04	36.5
5	7	MMDELM	1200	8.97E-01	7.47E-04	8.62	84.84	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00	6.5
6	16	INPVAL	1	6.25E-01	6.25E-01	6.01	90.85	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00	23.2
7	2	LTFADJ	1	2.91E-01	2.91E-01	2.80	93.65	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00	12.4
8	14	ADJTRI	1	2.50E-01	2.50E-01	2.41	96.06	6.10E+04	3.47E+04	8.71E+03	1.04E+05	21.14	8.8
9	12	SMBFAC	1	1.48E-01	1.48E-01	1.42	97.48	3.29E+03	3.93E+03	8.18E+02	8.03E+03	233.60	12.7
10	1	[illegible]	1	[illegible]	[illegible]	[illegible]							

Description	Million per second	Number	
Instructions Issued	MIPS = 34.32	818106881	4.49 Average CP per instruction
Clock periods holding issue		2455223733	66.84% of CP holding issue
Instruction Buffer Fetches	0.20	4668061	
I/O memory references	0.17	4057847	0.21% of total references
CPU memory references	82.13	1957752220	0.77 Memory references per FLO
Floating point add operations	54.02	1287758306	50.50% of total FLOPS
Floating point multiply operations	52.95	1262155085	49.49% of total FLOPS
Floating point reciprocal operations	0.01	301687	0.01% of total FLOPS
Total floating point operations	MFLOPS = 106.98	2550215078	100.00% of program FLOPS

********* Group 0 - Condensed output for all routines *********

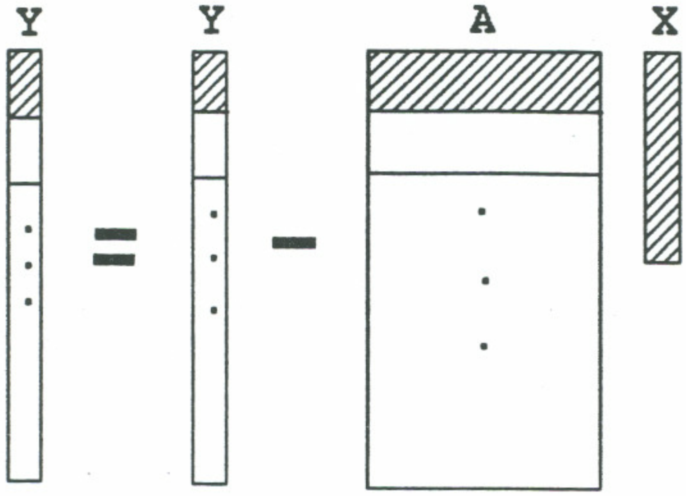
Rank	Order	Name	Times called	Time in seconds	Average Time	Execute Percent	Accum Percent	Adds	Mults	Recip	FLOPS	Memory /FLOP	Millio Mem/Se
1	20	SGMVI8	158258	5.75E+00	3.63E-05	24.13	24.13	7.29E+08	7.00E+08	0.00E+00	1.43E+09	0.73	180.4
2	8	MMDUPD	411	5.24E+00	1.27E-02	21.98	46.10	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00	6.6
3	19	SGEMV8	22740	4.07E+00	1.79E-04	17.08	63.18	5.42E+08	5.38E+08	0.00E+00	1.08E+09	0.64	168.5
4	18	GSICF2	1	3.41E+00	3.41E+00	14.31	77.50	6.10E+06	1.69E+07	2.16E+05	2.32E+07	4.91	33.4
5	7	MMDELM	8303	1.91E+00	2.30E-04	8.01	85.50	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00	7.0
6	16	INPVAL	1	1.11E+00	1.11E+00	4.67	90.17	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00	15.3
7	2	LTFADJ	1	5.99E-01	5.99E-01	2.51	92.68	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00	12.2
8	14	ADJTRI	1	5.52E-01	5.52E-01	2.32	95.00	2.47E+05	1.41E+05	3.53E+04	4.24E+05	11.05	8.4
9	12	SMBFAC	1	4.69E-01	4.69E-01	1.97	96.97	2.19E+04	2.72E+04	5.46E+03	5.45E+04	107.94	12.5
10	1	SSSISM	1	2.23E-01	2.23E-01	0.93	97.90	4.70E+01	0.00E+00	0.00E+00	4.70E+01	27412.70	5.7
11	25	GSBSLV	1	1.58E-01	1.58E-01	0.66	98.56	7.80E+06	5.27E+06	0.00E+00	1.31E+07	1.53	126.7
12	5	GENMMD	1	1.30E-01	1.30E-01	0.55	99.11	8.52E+05	9.56E+04	8.71E+03	9.57E+05	2.61	19.2
13	13	RQSORT	5468	6.18E-02	1.13E-05	0.26	99.36	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00	20.4
14	24	GSFSLV	1	5.40E-02	5.40E-02	0.23	99.59	1.42E+06	1.42E+06	7.05E+02	2.84E+06	1.82	95.6
15	9	MMDNUM	1	5.08E-02	5.08E-02	0.21	99.80	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00	13.7
16	6	MMDINT	1	1.69E-02	1.69E-02	0.07	99.88	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00	25.1
17	4	ICOPY	6	1.16E-02	1.93E-03	0.05	99.92	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00	280.3
18	10	MMDPAR	1	1.03E-02	1.03E-02	0.04	99.97	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00	28.1
19	17	SSSFCT	1	4.99E-03	4.99E-03	0.02	99.99	5.80E+01	0.00E+00	0.00E+00	5.80E+01	19762.36	229.4
20	26	SSSPST	1	9.46E-04	9.46E-04	0.00	99.99	3.00E+01	0.00E+00	0.00E+00	3.00E+01	377.77	11.9
21	23	GSSOLV	1	9.10E-04	9.10E-04	0.00	100.00	8.00E+00	1.07E+05	3.56E+04	1.42E+05	0.75	117.8
22	22	SSCTR	2	7.98E-04	3.99E-04	0.00	100.00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00	267.7
23	11	SSSSYM	1	5.23E-05	5.23E-05	0.00	100.00	1.18E+02	0.00E+00	0.00E+00	1.18E+02	7.76	17.5
24	3	SSSORD	1	5.19E-05	5.19E-05	0.00	100.00	8.00E+01	0.00E+00	1.00E+00	8.10E+01	11.00	17.1
25	21	SSSSLV	1	3.17E-05	3.17E-05	0.00	100.00	6.30E+01	0.00E+00	0.00E+00	6.30E+01	[illegible]	[illegible]

Rank	Order	Name	Times called	Time in seconds	Average Time	Execute Percent	Accum Percent	Adds	Mults	Recip	FLOPS	Memory /FLOP	Million Mem/Sec
1	8	MMDUPD	200	1.19E+00	5.95E-03	39.51	39.51	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00	6.55
2	17	GSICF2	1	4.92E-01	4.92E-01	16.31	55.82	1.16E+06	2.48E+06	2.63E+04	3.66E+06	4.36	32.49
3	19	SGMVI8	19005	4.11E-01	2.16E-05	13.64	69.46	5.39E+07	5.05E+07	0.00E+00	1.04E+08	0.75	190.71
4	7	MMDELM	1288	2.95E-01	2.29E-04	9.80	79.26	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00	7.14
5	18	SGEMV8	3509	2.05E-01	5.84E-05	6.80	86.06	2.77E+07	2.71E+07	0.00E+00	5.47E+07	0.65	173.99
6	12	SMBFC2	1	1.13E-01	1.13E-01	3.75	89.81	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00	8.61
7	15	INPVAL	1	1.10E-01	1.10E-01	3.64	93.46	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00	13.91
8	2	LTFADJ	1	6.01E-02	6.01E-02	1.99	95.45	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00	12.22
9	13	ADJTRI	1	5.62E-02	5.62E-02	1.86	97.32	2.73E+04	1.54E+04	3.90E+03	4.66E+04	10.15	8.43
10	1	SSSISM	1	2.24E-02	2.24E-02	0.74	98.06	4.70E+01	0.00E+00	0.00E+00	4.70E+01	2767.34	5.80
11	5	GENMMD	1	2.19E-02	2.19E-02	0.73	98.78	1.25E+05	1.38E+04	1.49E+03	1.41E+05	2.92	18.82
12	24	GSBSLV	1	1.83E-02	1.83E-02	0.61	99.39	9.27E+05	6.47E+05	0.00E+00	1.57E+06	1.53	131.00
13	23	GSFSLV	1	6.49E-03	6.49E-03	0.22	99.61	1.47E+05	1.47E+05	1.12E+02	2.94E+05	1.80	81.51
14	9	MMDNUM	1	5.39E-03	5.39E-03	0.18	99.79	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00	13.97
15	6	MMDINT	1	1.87E-03	1.87E-03	0.06	99.85	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00	25.30
16	4	ICOPY	6	1.37E-03	2.28E-04	0.05	99.90	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00	284.02
17	10	MMDPAR	1	1.36E-03	1.36E-03	0.05	99.94	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00	25.09
18	25	SSSPST	1	9.45E-04	9.45E-04	0.03	99.97	3.00E+01	0.00E+00	0.00E+00	3.00E+01	377.80	11.99
19	16	SSSFCT	1	5.14E-04	5.14E-04	0.02	99.99	5.80E+01	0.00E+00	0.00E+00	5.80E+01	1969.95	222.37
20	22	GSSOLV	1	1.14E-04	1.14E-04	0.00	99.99	8.00E+00	1.17E+04	3.95E+03	1.56E+04	0.79	107.72
21	21	SSCTR	2	7.42E-05	3.71E-05	0.00	100.00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00	321.46
22	11	SSSSYM	1	5.20E-05	5.20E-05	0.00	100.00	1.18E+02	0.00E+00	0.00E+00	1.18E+02	7.58	17.21
23	3	SSSORD	1	5.19E-05	5.19E-05	0.00	100.00	8.00E+01	0.00E+00	1.00E+00	8.10E+01	11.00	17.17
24	20	SSSSLV	1	3.19E-05	3.19E-05	0.00	100.00	6.30E+01	0.00E+00	0.00E+00	6.30E+01	8.97	17.70
25	14	SSSIVM	1	1.00E-05	1.00E-05	0.00	100.00	5.00E+01	0.00E+00	0.00E+00	5.00E+01	6.06	30.20
0	0	PROGRAM	1	3.01E+00	3.01E+00	100.00	100.00	8.40E+07	8.08E+07	3.58E+04	1.65E+08	0.90	49.06

ounters in this report were adjusted to
eflect the time used by Perftrace itself.

or each time a routine was called, the following
alues were subtracted from the counters before
isplay.

Instructions Issued: 320
CP Holding Issue: 792 *
Instruction Buffer Fetches: 16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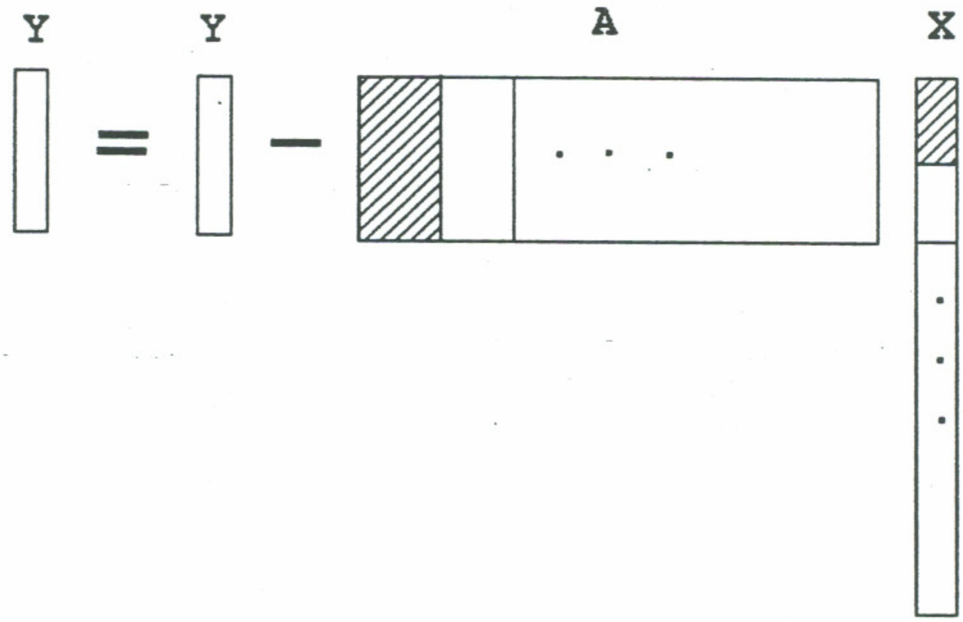

Y
Y
A
X

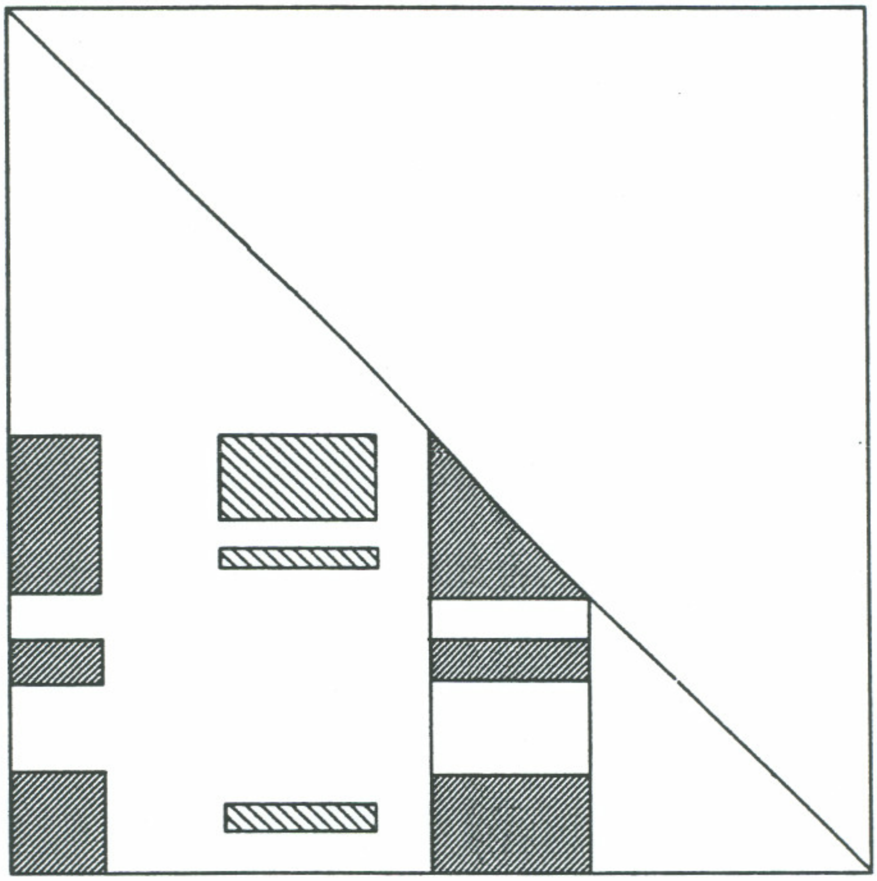
Numerical factorization - Autotasking results
Cray YMP - running 6.49ns

Problem	Neqns	NCPU=1 Mflops	NCPU=2 Mflops	NCPU=4 Mflops	NCPU=5 Mflops	NCPU=6 Mflops	NCPU=7 Mflops	NCPU=8 Mflops
BCSSTK15	3948	182.81	233.36	278.34	288.70	293.41	295.96	296.55
BCSSTK24	3562	128.86	146.08	155.46	156.31	156.39	157.33	155.93
BCSSTK31	35588	232.38	370.13	523.55	563.57	597.64	621.17	637.15
BCSSTK33	8738	238.86	381.52	548.33	588.90	621.74	644.78	663.06
NASASRB	54870	231.71	355.79	491.45	523.33	548.90	563.97	578.02
CUBE17	24565	280.75	547.77	1005.92	1202.64	1382.94	1545.12	1681.92
EG-AAT	25302	268.87						1184.77

Things to investigate:

- Relaxed supernode concept
- Supernode-supernode update
- Higher level parallelization at the supernode level

Supernode-Supernode Update



THE END